CS 381 – HW8 Solutions

1. Suppose $L \subseteq \Sigma^*$, $L' \subseteq \Delta^*$, We need to find a function $\sigma : \Sigma^* \to \Delta^*$, such that for all $x \in \Sigma^*$:

$$x \in L \Longleftrightarrow \sigma(x) \in L^{'}$$

Because L' is non-trivial, L' and L' complement are not empty. We can pick some $y_1 \in L'$ and $y_2 \notin L'$. And define σ in this way: for all $x \in \Sigma^*$

$$x \in L, \sigma(x) = y_1 \in L'$$

$$x \notin L, \sigma(x) = y_2 \notin L'$$

Now we only need to show that σ is total and computable. Since L is recursive, there exists some total Turing machine T computing L. Now we can construct a total Turing machine T' computing σ in this way: on input x, T' simulate T on input x, if T accepts x, T' accepts and writes y_1 on its tape. Or if T rejects x, T' rejects and writes y_2 on its tape. Thus σ is total and computable, which completes the proof.

2. Find a pair of languages L, L' for which $L \leq_m L'$ but $L' \nleq_m L$.

Solution 2: Take a language $L = \phi \in R$ and $L' \in R$ but L' non-trivial. By problem 1 we know that $L \leq_m L'$, however $L' \nleq_m L$ because L is trivial.

3. The claim is False. Prove by counterexample: Let A = HP, B = FIN, as defined in Kozen p.241. We have $A \leq_m B$ since $HP \leq_m FIN$. Now if the statement is true, then $\sim B \leq_m \sim A$ or equivalently $\sim FIN \leq_m HP$. But $\sim FIN$ is harder than $\sim HP$ which is in turn harder than HP. Thus we reach a contradiction.