2(i). Let L be a language defined by $0(0+1)^*$. Let (x,y) $\in$ R iff x $\in$ L and y $\in$ L. Claim: R is not left invariant, but is right invariant.

R is not left invariant. Consider x = 00, y = 01. Both x,y *in* L, therefore (x,y) *in* R. Let z = 1 $\in \Sigma^*$. Then neither zx, nor zy are in L. Therefore (zx, zy) not in R. Therefore, R is not left invariant.

R is right invariant. if (x,y) $\in$ R, x, y $\in$ L. Therefore we can represent x = 0x', y = 0y' where x',y' $\in \Sigma^*$. Then for any z $\in \Sigma^*$, xz = 0x'z = 0z', yz = 0y'z = 0z". Therefore xz, yz $\in$ R for any z. Therefore R is right invariant.

2(ii). Let L be a language defined by $(0+1)^*0$. Let (x,y) $\in$ R iff x $\in$ L and y $\in$ L. Claim: R is not right invariant, but is left invariant.

R is not right invariant. Consider x = 00, y = 10. Both x,y *in* L, therefore (x,y) *in* R. Let z = 1 $\in \Sigma^*$. Then neither xz, nor yz are in L. Therefore (xz, yz) not in R. Therefore, R is not right invariant.

R is right invariant. if (x,y) $\in$ R, x, y $\in$ L. Therefore we can represent x = x'0, y = y'0 where x',y' $\in \Sigma^*$. Then for any z $\in \Sigma^*$, zx = zx'0 = z'0, yz = zy'0 = z"0. Therefore zx, zy $\in$ R for any z. Therefore R is left invariant.

2(iii) Let L be a language defined by $(0+1)^*0(0+1)^*$. Let (x,y) $\in$ R iff x $\in$ L and y $\in$ L. Claim: R is both left and right invariant.

R is left and right invariant. if (x,y) $\in$ R, x, y $\in$ L. Therefore we c an represent x = x'0x", y = y'0y" where x', x", y',y" $\in \Sigma^*$. Then for any z,w $\in \Sigma^*$, zxw = zx'0x"w = z'0w', zyw = zy'0y"w = z"0w". Therefore zxw, zyw $\in$ R for any z, z'. Therefore R is both left and right invariant.

3) Find L(G) for the following grammar: G = ({A, B}, {a, b}, P, A), where P = {A $\rightarrow$ aBb, B$\rightarrow$bB, B$\rightarrow \epsilon$}. Prove your claim.

Claim: L(G)=$\{b^n a | n \in N, n > 0\}$, let M=$\{b^n a | n \in N, n > 0\}$
Proof.
L(G) $\subseteq$ M.
I will show: $B \rightarrow^* x \Rightarrow x = b^n$, for some n$\in$ N. We proceed by induction on the length of the G-derivation of x.

base case: $B \rightarrow x, x = \epsilon = b^0$
i.h.: $B \rightarrow^k x \Rightarrow x = b^k$
if $B \rightarrow^{k+1} x$, the derivation must be in this form: B$\rightarrow$bB$\rightarrow^k$x, then x can be written is this form: x=by and B$\rightarrow^k$y. By induction hypothesis, y=b^k,thus x=b^{k+1}.

for all x∈L(G), A→*x, the derivation must be like: A→bBa→*x, then x must be this form: x=bya and B→*y. As we have proved, y=b^n for some n∈N, thus x=$b^{n+1}a$ ∈M.

M ⊆ L(G).
for all x∈M, x=$b^n a$. We proceed by induction on n.
base case: n=1, x=ba, A→bBa→ba.
i.h.: if x=$b^k a$, A→*x
for x=b^{k+1}. by induction hypothesis, A→bBa→* $b^k a$, thus we have

$$\text{A}\rightarrow\text{bBa}\rightarrow\text{b(bBa)}\rightarrow^* b(b^k a) = b^{k+1}a.$$