CS 381 HW 4

Due Friday October 19th 2001

1) Let $M = (Q, \Sigma, q_0, \delta, F)$ be a DFA, and $k \in \mathbb{N}$. Define a relation E_k over Q as follows:

$$(p,q) \in E_k \text{ (i.e. } p \sim_{E_k} q) \text{ if for every } z \in \Sigma^* \text{ with } |z| > k,$$
$$\hat{\delta}(p,z) \in F \Leftrightarrow \hat{\delta}(q,z) \in F.$$

(i) Prove that E_k is an equivalence relation (i.e., that it is reflexive, transitive, and symmetric).

(ii) Describe the equivalence classes of E_0.

(iii) Prove, for all $p, q \in Q$, that if $p \sim_{E_k} q$ then $p \sim_{E_{k-1}} q$.

(iv) Prove that

$$p \sim_{E_{k+1}} q \iff$$
$$p \sim_{E_k} q \textit{ and } \forall \sigma \in \Sigma,\ \delta(p,\sigma) \sim_{E_k} \delta(q,\sigma)$$

2) Recall that a relation R over Σ^* is called *right-invariant* if $\forall\, w, w', z \in \Sigma^*$, if $(w, w') \in R$, then $(wz, w'z) \in R$. Similarly, we say that R is *left-invariant* if $\forall\, w, w'z, \in \Sigma^*$, if $(w, w') \in R$, then $(zw, zw') \in R$.

(i) Find a relation R over $\{0,1\}^*$ that is right-invariant but not left-invariant. Prove it!

(ii) Find a relation R over $\{0,1\}^*$ that is left-invariant but not right-invariant. Prove it!

(iii) Find a non-trivial relation over $\{0,1\}^*$ that is both left-invariant and right-invariant. Prove it!

3) Find $L(G)$ for the following grammar: $G = (\{A, B\}, \{a, b\}, P, A)$, where $P = \{A \to bBa,\ B \to bB,\ B \to \varepsilon\}$. Prove your claim.

4) For each of the following languages L, find a context-free grammar such that $L(G) = L$. Prove it.

(i) $L = \{w \in \{0,1\}^* \mid \#_0(w) = \#_1(w)\}$

(ii) $L = \{a^i b^j \mid i = 3j + 2\}$