CS 381 – HW3 Solutions 1,3,4

1i. Prove that the family of non-regular languages is closed under complementation.

Proof:
Suppose not. Then we can find some irregular L such that L^C is regular. But we know regular languages to be closed under complementation, which means $(L^C)^C = L$ must be regular. This is a contradiction, and thus irregular languages must be closed under complementation.

1ii. Show that the family of non-regular languages is not closed under the union operation.

Proof:
We know some irregular languages exist, so lets take any irregular L. L^C is also irregular from part i. $L \cup L^C = \Sigma^*$, which is regular. Thus irregular languages are not closed under union.

1iii. Prove that there is a family W of infinitely many non-regular languages such that $\bigcup\{L|L \in W\}$ is regular.

Proof:
We know that infinitely many irregular languages exist. Take W to be any infinite set of irregular languages, making sure W contains some language L and its complement L^C. The infinite union is Σ^*, which is regular.

3. For a word $w = o_1 \ldots o_n$, let $\overline{w}$ be the reverse word $\overline{w} = o_n \ldots o_1$. Prove that $L = \{w\overline{w}|w \in \{0,1\}^*\}$ is not a regular language.

Proof:
Lets use the pumping lemma / demon game. The demon gives us some n. We reply with word $x = 0^{n+1}110^{n+1}$ where clearly $|x| > n$ and also $x \in L$. The demon now partitions w into parts $xyz = w$ such that $|xy| \leq n+1$ and $|y| \geq 1$. We must show that regardless of the demons partitioning, the string w cannot be pumped. That is, we must find an $i \in \mathbb{N}$ such that $xy^iz \notin L$. Because of our clever choice of w, this is fairly easy. Any xy the demon picks can contain only 0s because the first $n+1$ digits of w are 0. Because y has non-zero length, we know $y = 0^j$ for some $j \geq 1$. Lets set $i = 0$ and $xy^iz = 0^{n+1-j}110^{n+1}$ which clearly is not in L. Therefore, L cannot be pumped. L is irregular.

4. Prove that $L = \{0^k1^n0^n \mid k,n > 0\} \cup \{1^i0^j|i,j \geq 0\}$ satisfies the pumping lemma.

Proof:
Lets take $n = 0$ and show that any string $w \in L, w \neq \epsilon$ can be pumped. If $w \in L$ either $w = 0^k1^n0^n$ $k,n > 0$ or $w = 1^i0^j$ $i,j \geq 0$. In the first case, choose

$x = \epsilon, y = 0, z = 0^{k-1}1^n0^n$. $|xy| = 1 \leq n + 1 = 1$ and we can pump because $xy^iz = 0^{i+k-1}1^n0^n$ and either $i+k-1 > 0$ and we are of the form $0^k1^n0^n$ $k, n > 0$ or $i + k - 1 = 0$ and we are of the form 1^i0^j $i, j \geq 0$. In the second case either $i > 0$ or $j > 0$ because $|w| \geq 1$. If $i > 0$ choose $x = \epsilon, y = 0, z = 0^{i-1}1^j$ wne we can clearly pump. If $i = 0$ then $j > 0$ so choose $x = \epsilon, y = 1, z = 1^{j-1}$ and once again we can easily pump. Thus L satisfies the pumping lemma.