

1. (10 points) Apply the minimization algorithm to the following DFA. Give the resulting minimal state DFA.

2. (15 points) Consider a CFG G having productions

$$S \longrightarrow XdY, \quad X \longrightarrow aX \mid \epsilon, \quad Y \longrightarrow bY \mid b.$$

- a) Find a Chomsky Normal form of G
- b) Find a Greibach Normal form of G
- c) Find a NPDA M such that $L(M) = L(G)$.

3. (10 points) Use the CKY algorithm to decide a membership problem for CFG G

$$S \longrightarrow aYX, \quad X \longrightarrow aX \mid b, \quad Y \longrightarrow Ya \mid b$$

and strings $x = abaab$, $y = ababa$. Solutions other than given by the CKY algorithm will bring only a partial credit.

4. (10 points) Find a CFG G equivalent to the NPDA M having transitions

$$\delta(s, a, \perp) = (s, \perp), \quad \delta(s, b, \perp) = (t, BB), \quad \delta(t, a, B) = (t, \epsilon).$$

If you use the official method from lectures (textbook), there is no need to justify the result. Otherwise, prove that $L(M) = L(G)$. You will be awarded up to 2 bonus points for bringing G to a concise form.

5. (15 points) Which of the following sets a) - c) are CFL and which are not. Give justification. (Hint: examples of languages which are known not to be CFL:

$$\{a^n b^n a^n \mid n > 0\}, \quad \{a^n b^{n^2} \mid n > 0\}, \quad \{x \mid \#a(x) = \#b(x) = \#c(x)\}, \quad \{xx \mid x \in \{a, b\}^*\}.$$

Here $\#l(x)$ denotes the number of letters l in a string x .

- a) the set of all strings x over $\{a, b, c, d\}$ such that $\#a(x) = \#b(x) = \#c(x) = \#d(x)$.
- b) all strings $\{a^n b^m c^n \mid m, n > 0\}$,
- c) the set of all strings x over $\{a, b\}$ such that $\#b(x) = (\#a(x))^2$.