CS381 Fall 2000 **Handout 32**
November 17, 2000

1. Reading: D. Kozen *Automata and Computability*, lecture 35
J. Hopcroft and J. Ullman *Introduction to Automata Theory, etc.*, section 8.6.

2. The main message of this lecture:

> **Rice Theorem does not cover decidability questions in CFG; one has to do reductions from known undecidable problems (e.g. the Halting Problem) by hand.**

Some problems in CFGs are decidable: nonemptiness, finiteness of the generated CFL, etc. (cf. HO25). With the general theory of computability at hand one can approach undecidability questions in CFGs. The paradigmatic example of such a question is the problem of deciding whether a given CFG generates all strings.

A canonical method of establishing undecidability of a problem P is building a reduction from the halting problem to P. In particular, one could try to emulate Turing machine computations in P in such a way that any decision algorithm for P would immediately give a decision procedure for the Halting Problem. Since the latter in known to be undecidable, the former is undecidable as well. This trick worked well in the proof of the Rice Theorem.

Theorem 32.1. *It is impossible to decide for a given CFG whether it generates all strings.*

Proof follows closely the one from Lecture 35 in Kozen's book.

Homework problems. Kozen p.311, No.3; p.343, No.108b.