CS 381 Fall 2000
Solutions to Homework 3

Handout #6

Problem 1

Let the states of the given *NFA* be $0, 1$ respectively. Then the states of the DFA will be a subset of the powerset of {0,1}. The transition function δ of the DFA is given by :

$$\delta(\{0\}, a) = \{0, 1\}$$
$$\delta(\{0\}, b) = \emptyset$$
$$\delta(\{0, 1\}, a) = \{0, 1\}$$
$$\delta(\{0, 1\}, b) = \{0, 1\}$$

The start state is $\{0\}$ and both the states are final states.

Problem 2

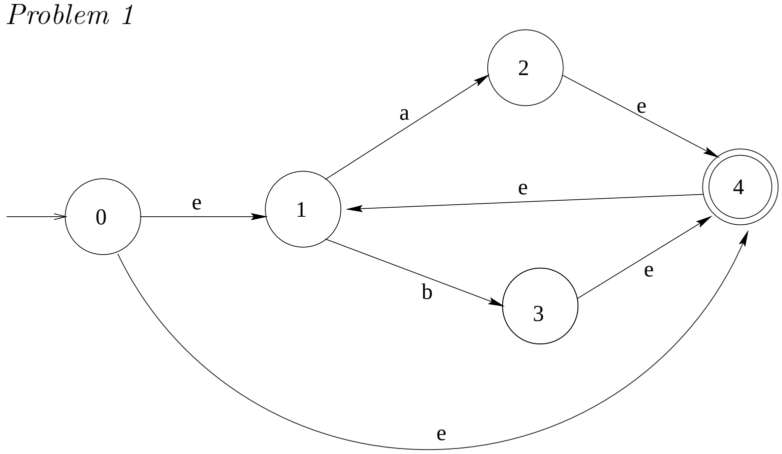
The states of the DFA are a subset of the powerset of {s,t,u,v}. The transition function δ of the DFA is given by :

$$\delta(\{s\}, a) = \{s, t\}$$
$$\delta(\{s\}, b) = \{s\}$$
$$\delta(\{s, t\}, a) = \{s, t, u\}$$
$$\delta(\{s, t\}, b) = \{s\}$$
$$\delta(\{s, t, u\}, a) = \{s, t, u\}$$
$$\delta(\{s, t, u\}, b) = \{s, v\}$$
$$\delta(\{s, v\}, a) = \{s, t\}$$
$$\delta(\{s, v\}, b) = \{s\}$$

The start state is $\{s\}$ and $\{s, v\}$ is the final state.

Handout #7

Problem 1



Computation of $\epsilon{-}closure$:

$$\epsilon{-}closure(0) = \{0, 1, 4\}$$
$$\epsilon{-}closure(1) = \{1\}$$
$$\epsilon{-}closure(2) = \{2, 4, 1\}$$
$$\epsilon{-}closure(3) = \{3, 4, 1\}$$
$$\epsilon{-}closure(4) = \{4, 1\}$$

Computation of δ^+ :

$$\begin{aligned}
\delta^+(0,a) &= \{2\} \\
\delta^+(0,b) &= \{3\} \\
\delta^+(2,a) &= \{2\} \\
\delta^+(3,b) &= \{3\} \\
\delta^+(2,a) &= \{2\} \\
\delta^+(3,b) &= \{3\}
\end{aligned}$$

0 is the start state and all the three states $(0, 2, 3)$ are final states.

Problem 2

(a) $(b + ab^*a)^*$

(b) $a^*b(a + ba^*b)^*$

(c) $(b + ab^*a)^* + a^*b(a + ba^*b)^*$

(d) It is too hard to guess the solution, thus, we have to solve the problem in a sy stematic way. First we build a (D)FA for the language

$$L = \{w \mid w\text{contains even number of } \mathtt{a}\text{'s and odd number of } \mathtt{b}\ \text{'s}\}$$

(0 is the starting and 2 is the final state).

	a	b
0	1	2
1	0	3
2	3	0
3	2	1

Then we use the standard construction to build a regular expresssion from FA (we guessed $\alpha_{i,j}^{0,1,2}$, so we did not need to do the complete construct ion)

$$\alpha_{0,2}^{\{0,1,2,3\}} = \alpha_{0,2}^{\{0,1,2\}} + \alpha_{0,3}^{\{0,1,2\}} \left(\alpha_{3,3}^{\{0,1,2\}}\right)^* \alpha_{3,2}^{\{0,1,2\}}$$

Let $X = (aa + bb)$.

$$\begin{aligned}
\alpha_{0,2}^{\{0,1,2\}} &= X^*b \\
\alpha_{0,3}^{\{0,1,2\}} &= X^*(ab + ba) \\
\alpha_{3,3}^{\{0,1,2\}} &= \epsilon + (ab + ba)X^*(ab + ba) \\
\alpha_{3,2}^{\{0,1,2\}} &= a + (ba + ab)X^*b
\end{aligned}$$

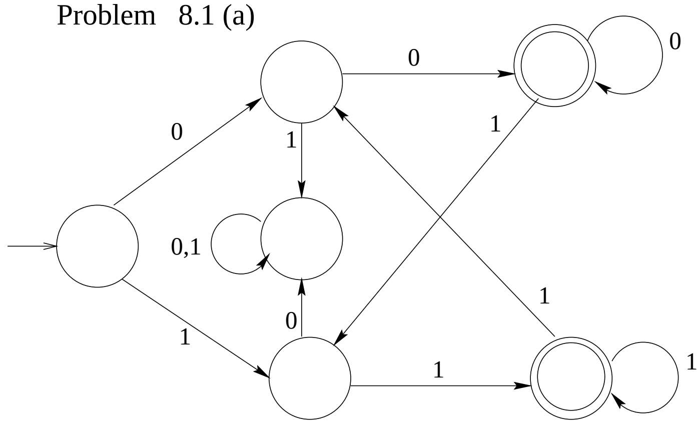
Putting together and simplifying:

$$\alpha_{0,2}^{\{0,1,2,3\}} = X^* \left[b + (ab + ba)\left[(ab + ba)X^*(ab + ba)\right]^* (a + (ba + ab)X^*b)\right]$$

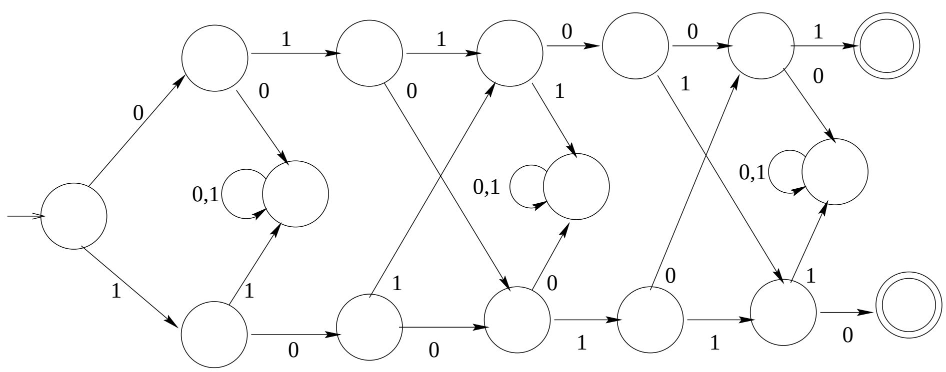
Handout #8

Problem 1

Problem 8.1 (a)



Problem 8.1(b)



Problem 8.1 (c)