CS 322: Final

Tuesday, August 6, 2002

Print Name________________________________

Signature__________________________________

Student ID Number__________________________

The rules for the final are as follows:

- **Write your name and student ID number on the front page of the exam booklet. Initial each of the remaining pages in the upper-right hand corner. Sign the front of the exam booklet. Your exam will not be graded if you have not signed the front page of the booklet.**
- The exam has 8 questions and 1 extra-credit question. The exam is 10 pages long (including this page). Be sure you have all of the pages before beginning the exam.
- This exam will last for 2 hours.
- Show **ALL** work for partial/full credit. This includes any definitions, mathematics, figures, etc.
- The exam is closed book and closed notes, and no calculators are allowed on the exam.
- No collaboration of any kind is allowed on the exam.

1. ________ (20 points) 5. ________ (20 points)

2. ________ (20 points) 6. ________ (20 points)

3. ________ (20 points) 7. ________ (20 points)

4. ________ (20 points) 8. ________ (10 points)

TOTAL: ________ (150 points)

1. (20 points)

   (a) (2 points) Describe a real-world application where interpolation is used.

   (b) (2 points) Describe a real-world application where linear algebra arises.

   (c) (2 points) Describe a real-world application where optimization is used.

   (d) (2 points) Name the one area of scientific computing that we touched upon briefly that was not covered in your textbook. (Hint: Think back to PS 1.)

   (e) (2 points) Name the tool from calculus that was frequently used to derive the error terms for many of the numerical methods in the text.

   (f) (4 points) How are vector norms used in scientific computing?

   (g) (4 points) What does the condition number of a matrix tell us?

   (h) (2 points) Name your favorite area of scientific computing.

2. (20 points)

   (a) (10 points) Find the constants x_0, x_1, and c_1 so that the quadrature formula

   $$\int_0^1 f(x)dx = \frac{1}{2}f(x_0) + c_1 f(x_1)$$

   is exact for polynomials of as high a degree as possible.

   (b) (10 points) Write a Matlab script that uses fzero to find the unique fixed point of $f(x) = 0.5(\sin(x) + \cos(x))$. Use the particular calling sequence that allows you to provide an interval of the root, and justify your choice of the initial interval. (Recall that the calling sequence is fzero('f',x0), where x0 is a column 2-vector such that f(x0(1)) and f(x0(2)) have opposite signs.)

3. (20 points)

(a) (10 points) Which method should be used to solve each of the following problems? (Base your decision upon efficiency.) Give the name of the method and completely justify your choice.

i. Solve $Ax = b$, where $A = \begin{pmatrix} 9 & -6 \\ -3 & -5 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$.

ii. $\min \|Ax - b\|_2$, where $A = \begin{pmatrix} 2 & 6 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 10 \\ 1.5 \\ 7 \end{pmatrix}$.

iii. Solve $Ax = b$, where $A = \begin{pmatrix} 10 & -1 & -2 \\ -1 & 9 & 0 \\ -2 & 0 & 7 \end{pmatrix}$ and $b = \begin{pmatrix} 26 \\ 15 \\ 1 \end{pmatrix}$.

(b) (10 points) Discuss 3 issues that arise in interpolation.

4. (20 points)

(a) (10 points) Write a Matlab script that uses the Gauss-Newton Method to find the value of λ such that $exp(\lambda x)$ is the nonlinear least-squares approximation to $(0, 0)$, $(0.25, 1.2840)$, $(0.5, 1.6487)$, $(0.75, 2.1170)$, and $(1.0, 2.7183)$. Use $\lambda_0 = 1$ as a starting point. Your loop should run until two successive values of λ are within 0.001 of each other or until 20 iterations have been executed. **You may NOT use any pre-existing Matlab functions for this problem.**

(b) (10 points) Discuss the tradeoffs of Runge-Kutta versus Adams-Moulton methods for integrating an IVP.

5. (20 points)

   (a) (10 points) Write down a specialized efficient Matlab function for back-substitution to solve the linear system $Ax = b$, where A is a bidiagonal upper triangular matrix. Analyze the number of flops required by your algorithm (accurate to the leading term). **You may not use any pre-existing Matlab functions to solve this problem.**

   (b) (10 points) Given a concrete example of when it would be desirable to use Hermite cubic interpolation rather than cubic spline interpolation. (Hint: consider monotonic data.) You do NOT have to compute any interpolants; simply draw the interpolants and explain your example.

6. (20 points)

(a) (10 points) Show how to construct a rotation of the form

$$G = \begin{pmatrix} c & s \\ -s & c \end{pmatrix},$$

where $c = \cos(\theta)$, and $s = \sin(\theta)$, such that Gx is of the form

$$\begin{pmatrix} 0 \\ y_2 \end{pmatrix}$$

for any column 2-vector x. (This should be done mathematically, not with Matlab code.)

(b) (10 points) Convert the following integral into one to which Gaussian quadrature could be applied:

$$\int_1^{1.5} e^{-x^2}.$$

You do NOT need to approximate the integral using Gaussian quadrature.

7. (20 points)

(a) (15 points) Write a Matlab script that uses Newton's Method to approximate, to within 10^{-4}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$. **You may not use any pre-existing Matlab functions on this problem.**

(b) (5 points) Name one advantage that Newton interpolation has over Vandermonde interpolation and one advantage that Vandermonde interpolation has over Newton interpolation.

8. (10 points)

(a) Consider solving the IVP in the special form $\frac{dy}{dt} = Ay + b, y(0) = y_0$ using the Backward-Euler Method. Here A is an $n \times n$ matrix, b is a column n-vector, and $y(t) = \begin{pmatrix} y_1(t) \\ \vdots \\ y_n(t) \end{pmatrix}$. Derive an explicit formula for y^{k+1} and indicate how y^{k+1} should be determined computationally. (Here y^k denotes the kth iterate.) **Efficiency matters.**

EXTRA-CREDIT (5 points)

Give a function $f(x)$ that has exactly 1 root at 0 and that when Newton's Method is applied to $f(x)$ with any nonzero starting point it alternates between two values, thus, never converging to the root.