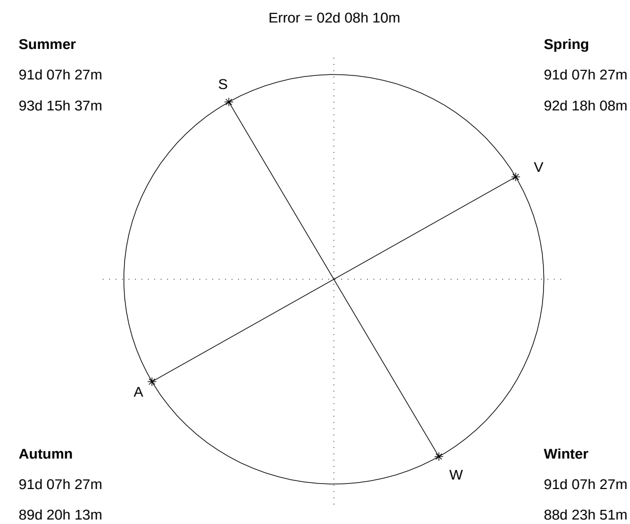
CS 322: Assignment P5

Due: Wednesday, May 1, 2002 in lecture or Friday, May 3, in Upson 4130 by 4pm.

You may work in pairs. Follow the course rules for the submission of assignments. Do not submit work unless you have adhered to the principles of academic integrity as descibed on the course website. Points will be deducted for poorly commented code, redundant computation that seriously effects efficiency, and failure to use features of MATLAB that are part of the course syllabus.

Part A (10 pts) Season Lengths and the Eccentric Model

Suppose the Sun moves counterclockwise along the circle $x^2 + y^2 = 1$ at a uniform angular rate. Assume that the Earth is located at (0,0) and has a 23.5^{O} axis tilt with the tilt being towards the summer solstice point S in the following figure:



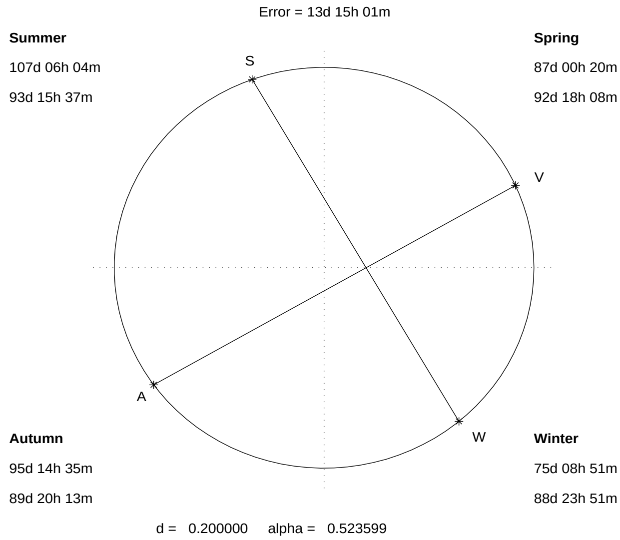
The points A, W, and V are respectively the autumnal equinox point, the winter solstice point, and the vernal equinox point. It follows that spring, summer, fall, and winter begin when the Sun passes through V, S, A, and W. As reported in the figure, we have a problem because the model predicts that each season has length $365.2425/4 = 91$ days + 7 hours + 21 minutes when in fact they have rather different durations:

Season	Start	Length
Spring	19:02 March 20, 2002	92d 18h 08m
Summer	13:07 June 21, 2002	93d 15h 37m
Autumn	04:29 September 23, 2002	89d 20h 13m
Winter	01:01 December 22, 2002	88d 23h 51m
Spring	00:31 March 21, 2003	- - - - - - - - -

This will be the case no matter how we "tilt" the equinox/solstice "crosshair".

In order to improve the predictive power of the model we continue to move the Sun uniformly along the circle. However, we now position the Earth at the point $(d, 0)$. Assume that $V = (\cos(\alpha), \sin(\alpha))$. With this

2-parameter model we can vary the lengths of the predicted seasons. For example, by setting $d = .2$ and $\alpha = \pi/6$ we obtain



By moving the Earth to (.2,0) we have increased the predicted length of summer (good) and autumn (not good) and decreased the predicted length of winter (good) and spring (not good). The question before us is how to best choose d and α so as to minimize

$$\phi(d,\alpha) = \max\{\ |T_V - \hat{T}_V|\ ,\ |T_S - \hat{T}_S|\ ,\ |T_A - \hat{T}_A|\ ,\ |T_W - \hat{T}_W|\ \}$$

i.e., the maximum discrepancy between the actual season lengths T_V, T_S, T_A, and T_W and their predicted counterparts $\hat{T}_V$, $\hat{T}_S$, $\hat{T}_A$, and $\hat{T}_W$.

Our approach will be to use the MATLAB one-dimensional minimizer `fmin` alternately applied to the functions

$$f_\alpha(d) = \phi(d,\alpha) \qquad \alpha \text{ fixed}$$

and

$$f_d(\alpha) = \phi(d,\alpha) \qquad d \text{ fixed}$$

In particular, write a script `P5A` that initializes both d_0 and α_0 to zero and then repeats the following for $i = 1{:}6$

- Use `fmin` to compute d_i so that $f_{\alpha_{i-1}}(d_i)$ is the minimum value of $f_{\alpha_{i-1}}$ across the interval $[-1,1]$.
- Display the results using `DrawEccentric`. (See below.)
- Use `fmin` to compute α_i so that $f_{d_i}(\alpha_i)$ is the minimum value of f_{d_i} across the interval $[0,2\pi]$.
- Display the results using `DrawEccentric`. (See below.)

A function

```
  function [x,y] = CirclePoints(d,alpha)
% Determines the equinox and solstice points for the eccentric model
% with displacement d and rotation alpha.
%
% x and y are row 4-vectors with the property that
%
%  (x(1),y(1)) = coordinate of the Vernal   Equinox  point
%  (x(2),y(2)) = coordinate of the Summer   Solstice point
%  (x(3),y(3)) = coordinate of the Autumnal Equinox  point
%  (x(4),y(4)) = coordinate of the Winter   Solstice point
```

is provided on the website. You may find it handy to write a function `sE = SeasonLength(d,alpha)` that returns a 4-vector of season lengths. This makes it easy write f_α and f_d. Use

$$T_V = 92.7556 \qquad T_S = 93.6507 \qquad T_A = 89.8424 \qquad T_W = 88.9938$$

and a total year length $Y = 365.2425$. Note that if $\overline{VS}$ is the arclength from V to S then $\hat{T}_V = Y \cdot \overline{VS}/(2\pi)$. To compute the arc length you might want to work with the arcsin function `asin` and the isoceles triangle defined by (0,0), V and A. Obtain similar expressions for the other predicted season lengths.

Your script `P5A` should call

```
  function DrawEccentric(d,alpha,sE,sA)
% Displays the eccentric model with displacement d and rotation
% alpha.
% sE is a row 4-vector of estimated season lengths (in days).
% sA is a row 4-vector of actual season lengths (in days).
% Both sE and sA specify these lengths in Spring-Summer-Fall-Winter order.
```

after each d update and α update. Submit the figures associated with (d_1, α_1),(d_2, α_2), and (d_6, α_6). The process pretty much converges after six iterations. Also submit a listing of all scripts and functions that you had to write to produce these figures.

Part B (10 pts) Apollo 13

Consider the following IVP:

$$\begin{aligned}
\ddot{x}(t) &= 2\dot{y}(t) + x(t) - \frac{\mu_*(x(t)+\mu)}{r_1^3} - \frac{\mu_(x(t)-\mu_*)}{r_2^3}, \qquad & x(0) = 1.2 \quad \dot{x}(0) = 0, \\
\ddot{y}(t) &= -2\dot{x}(t) + y(t) - \frac{\mu_* y(t)}{r_1^3} - \frac{\mu y(t)}{r_2^3}, \qquad & y(0) = 0 \qquad \dot{y}(0) = -1.0493575,
\end{aligned}$$

where $\mu = 1/82.45$, $\mu_* = 1 - \mu$, and

$$\begin{aligned}
r_1 &= \sqrt{((x(t)+\mu)^2 + y(t)^2} \\
r_2 &= \sqrt{((x(t)-\mu_*)^2 + y(t)^2}
\end{aligned}$$

It describes the orbit of a spacecraft that starts behind the Moon (located at $(1-\mu, 0)$), swings by the Earth (located at $(-\mu, 0)$), does a large loop, and returns to the vicinity of the Earth before returning to its initial position behind the Moon at time $T_0 = 6.19216933$. Here, $\mu = 1/82.45$.

(a) Apply `ode45` with $t_{initial} = 0$, $t_{final} = T_0$, and $tol = 10^{-6}$. Plot the orbit twice, once with the default "pen" and once with `'.'` so that you can see how the time step varies. **(b)** Using the output from the `ode45` call in part (a), plot the distance of the spacecraft to Earth as a function of time across $[0, T_0]$. Use `spline` to fit the distance "snapshots." To within a mile, how close does the spacecraft get to the Earth's surface? Assume that the Earth is a sphere of radius 4000 miles and that the Earth-Moon separation is 238,000 miles. Use `fmin` with an appropriate spline for the objective function. Note that the IVP is scaled so that one unit of distance is 238,000 miles. **(c)** To the nearest minute, compute how long the spacecraft is hidden to an observer on earth as it swings behind the Moon during its orbit. Assume that the observer is at $(-\mu, 0)$ and that the Moon has diameter 2160 miles. Make intelligent use of `fzero`.

Submit output and all the scripts and functions that were required to produce it.