Supplemental questions - these must be turned in by Monday, 12/8/03, if you would like to have them graded before the final.

8. Instead of using truth tables to compute the value of a formula, an arithmetic procedure can be used. The basis for this approach is the representation of the basic components as follows:

Formula	Arithmetic
$\neg P$	$1 + P$
$P \wedge Q$	$P + Q + PQ$
$P \vee Q$	PQ
$P \rightarrow Q$	$(1 + P)Q$
$P \leftrightarrow Q$	$P + Q$

When the values T or F are assigned to a prime component P in a formula, the values 0 or 1 respectively are assigned to P in the arithmetic expression. (Yes, this really is T valued at 0 and F valued at 1.) The computations are performed in $\mathbb{Z}_Z$ so that $1 + 1 = 0$. Use this approach to prove the following tautologies:

$$\models A \wedge (A \rightarrow B) \rightarrow B$$

$$\models (A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$$

$$\models (A \wedge B) \leftrightarrow \neg(\neg A \vee \neg B)$$

9. (a) Give a sequence of deductive steps to prove that $A \rightarrow (B \rightarrow C)$, $\neg D \vee A$, $B \models D \rightarrow C$.

   (b) Give a derivation proof of "Art is a boy who does not own a car. Jane likes only boys who own cars. Therefore, Jane does not like Art." (Use $B(x)$, $O(x)$, $L(x,y)$ as predicates and a, j as constants.)

10. By exploiting 'satisfiability' demonstrate the validity or invalidity of the following arguments.

   (a) If Cornell wins then Ithaca will celebrate, and if Harvard wins then Boston will celebrate. Either Cornell will win or Harvard will win. However, if Cornell wins then Boston will not celebrate and if Harvard wins then Ithaca will not celebrate. So Ithaca will celebrate if and only if Boston does not celebrate.

   (b) If I take the bus and the bus is late I'll miss my appointment. If I miss my appointment and start to feel downcast then I should not go home. If I don't get that contract then I'll start to feel downcast and should go home. Therefore if I take the bus and the bus is late, I will get that contract.

   (c) If Smith wins the nomination he will be happy, and if he is happy he is not a good campaigner. But if he loses the nomination he will lose the confidence of the party. He is not a good campaigner if he loses the confidence of the party. If he is not a good campaigner then he should resign from the party. Either Smith wins the nomination or he loses it. Therefore he should resign from the party.