CS 280 Solution Set

Homework 7

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1(a). 300 Bernoulli Trials. Probability of success p=1/6. We know that

E (# of successes)=$E(Y_6)$=np (see book p.277)

$E(Y_6)$ = np = 300×1/6 **= 50**

1(b). $V(Y_6)$=npq (see book p.282)

=300×1/6×5/6=**125/3**

1(c). $E(Y_1 + Y_2+ Y_3) = E(Y_1) + E(Y_2) + E(Y_3)$= 50 + 50+ 50 =**150**

1(d). Make random variables X_1, X_2 ….. X_{300} .

X_i=1 if trial i yields 1,2,3 . X_i=0 otherwise.

$Y_1+Y_2+Y_3 = X_1 + X_2 \ldots\ldots X_{300}$

Because Xi are all pairwise independent(different trials):

$V(Y_1+Y_2+Y_3)= V(X_1) + V(X_2) +\ldots\ldots. +V(X_{300})$

$V(X_i)=E(X_i^2)-(E(X))^2=(1/2\times1^2 + 1/2\times0^2)-(1/2\times1+ 1/2 \times0)^2=1/2-1/4=1/4$

$V(Y_1+Y_2+Y_3)$ = 300×1/4 **= 75.**

2(a). $E(Y_1)= E(Y_2)\ldots..=E(Y_6)$

$E(X)= E(Y_1)+2E(Y_2) \ldots\ldots + 6E(Y_6)$=21×50 = **1050**

2(b). Make random variables X_1, X_2 ….. X_{300} . X_i = k if trial i yields k.

$X=X_1+ X_2 \ldots\ldots +X_{300}$

Because X_i are all pairwise independent (depend on different trials)

$V(X)= V(X_1) +V(X_2)+\ldots..+V(X_{300})$

$V(X_i)=1/6 \times (1+4+9\ldots.36) - (1/6\times (1+2+3\ldots+6))^2= 35/12$

$V(X)=300 \times V(X_i)$ = **875**

3. For each fruit, we use bars and stars to divide the fruit amongst 4 people. Therefore we get:

   Number of ways to split up the fruits= The number of ways to split up apples $\times$ number of ways to split up bananas $\times$ number of ways to split up pears=

   7!/(3! $\times$4!) * 6!/(3! $\times$3!) * 5!/(3! $\times$2!)= 7000

4(a). Two ways to do this:

Solution1:
Take Computer science books to be |, and non Computer Science books to be *. Since there are 5 bars, there are 6 bins in which we can place the 14 *'s. The middle 4 bins (between pairs of |'s) must have at least one * each in them. So, after placing 4 *'s in the middle 4 bins, we now have 10 *'s left to place in the 6 bins. The number of ways to place 10 stars in 6 bins is C(15,5). Each arrangement of *'s and |'s is a choice of CS book positions.

There are C(15,5) ways to choose the CS book positions. Now, for each choice of positions, lets permute the CS books by multiplying by 5!, and lets permute the non CS books by multiplying by 14!.

So our answer is:

C(15,5) $\times$ 5! $\times$ 14! = (15! $\times$ 14!) / 10!

Solution 2:

Take Computer science books to be |, and non Computer Science books to be *. The number of ways we can arrange those is the number of ways we can arrange the bars and stars such that there is at least one star between each two bars. One way to do this would be to make 4 Bar-Star pairs (|*), lets denote those by _. We then have 10 *'s, 4 _'s, and 1 |. We know the 4 _'s have to come before the |, so choosing the position of the CS books involves C(15,5). This is the number of ways we can choose 5 positions for 4 _'s and a |, in that order.

There are C(15,5) ways to choose the CS book positions. Now, for each choice of positions, lets permute the CS books by multiplying by 5!, and lets permute the non CS books by multiplying by 14!.

So our answer is:

$C(15,5) \times 5! \times 14! = (15! \times 14!) / 10!$

4(b). divide by the number of permutations of each indistinguishable group.
We get:

$(15! \times 14!) / (10! \times 8! \times 6! \times 5!)$

5. Inclusion-Exclusion:

A_k= Integers less than 4445 divisible by k

$|A_k| = \lfloor 4444/k \rfloor$

A= integers divisible by 2,3,5 or 7

$|A| = |A_2| + |A_3| + |A_5| + |A_7| - |A_2 \cap A_3| - |A_2 \cap A_5| - |A_2 \cap A_7| - |A_3 \cap A_5| - |A_3 \cap A_7| - |A_5 \cap A_7| + |A_2 \cap A_3 \cap A_5| + |A_2 \cap A_3 \cap A_7| + |A_2 \cap A_5 \cap A_7| + |A_3 \cap A_5 \cap A_7| - |A_2 \cap A_3 \cap A_5 \cap A_7|$

$= |A_2| + |A_3| + |A_5| + |A_7| - |A_6| - |A_{10}| - |A_{14}| - |A_{15}| - |A_{21}| - |A_{35}| + |A_{30}| + |A_{42}| + |A_{70}| + |A_{105}| - |A_{210}|$

= 2222 + 1481+ 888+634 –740 – 444 – 317 – 296 – 211 – 126 + 148 + 105 + 63 + 42 – 21 = **3428**

6(a) Permutations of the beads (taking into account that yellow are identical)= 23!/20!
Divide by 23 to account for rotation (23 rotations)
Also divide by 2 to account for mirror image of the permutation(turn over).

# of different necklaces= $23!/(20! \times 23 \times 2) = ((22 \times 21)/2)$ = **231**

6(b) Permutations of the beads (taking into account that yellow are identical and black are identical)= $23!/(20! \times 3!)$
Divide by 23 to account for rotation (23 rotations) to get:

$22!/(20! \times 3!) = 77$

Now, this is where it gets tricky. If this were like 6a, we would divide by 2 to account for the fact that we are double counting each necklace by counting both itself and its reflection. However, that is not the case here. Some necklaces are symmetric; that is they are not double counted because they have no unique reflection (the reflection of a symmetric necklace is simply itself). A symmetric necklace has one black bead at the center, with the two other black beads equidistant from the center. The number of such necklaces is 11 (the black beads on the sides can have distances 0-10 from the central bead).

Therefore, we have 77 necklaces not accounting for reflections. 11 of those are symmetric, and 66 are asymmetric. We know we double count the asymmetric necklaces only, which means the number of unique asymmetric necklaces is 33.

Total number of unique necklaces= 33+11 = **44**