CS280 Solutions
Homework 6
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Part A

1. Two dice are rolled. There are 36 total possible rolls. Let (x, y) represents a roll with value x on the first die, and value y on the second die.

a) Find the probability that the sum is divisible by 5.

P(sum divisible by 5) = P(sum = 5) + P(sum=10)
Dice rolls that give a sum of 5: {(1, 4), (2, 3), (3, 2), (4, 1)}
P(sum=5) = 4/36
Dice rolls that give a sum of 10: {(4, 6), (5, 5), (6, 4)}
P(sum=10)= 3/36.
P(sum divisible by 5)= **7/36**.

b) Find the probability that the sum is divisible by 3

P(sum divisible by 3) = P(sum=3) + P(sum=6) + P(sum=9) + P(sum=12).
Dice rolls that give a sum of 3 : {(1, 2), (2, 1)}
P(sum =3) = 2/36
Dice rolls that give a sum of 6: {(1, 5) , (2, 4), (3, 3), (4, 2), (5, 1)}
P(sum =3) = 5/36
Dice rolls that give a sum of 9: {(3, 6), (4, 5), (5, 4), (6, 3)}
P(sum =3) = 4/36
Dice rolls that give a sum of 12: {(6, 6)}
P(sum =3) = 1/36
P(sum divisible by 3)=12/36=**1/3**.

c) Find the probability that the difference is 1.

Rolls that have a difference of 1 : {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6) , (2, 1), (3, 2), (4, 3) , (5, 4), (6, 5)}
P(difference is 1) = 10/36 = **5/18**.

d) Find the probability that the difference is even.

The numbers on the two dice will have an even difference if both dice have odd numbers or both dice have even numbers.
P(difference is even)
= P(roll 1 is odd) * P(roll 2 is odd) + P(roll 1 is even) * P(roll 2 is even)
= 1/2 * 1/2 + 1/2 * 1/2
= **1/2**

2. A positive integer n less than 1001 is picked. The range of numbers is from 1 to 1000 inclusive. Assume the number is chosen randomly with a uniform distribution from the range.
a) Find the probability that the sum is divisible by 7.

There are floor(1000/7) = 142 positive integers in the range divisible by 7.
P(n divisible by 7) = 142/1000 = **71/500**.

b) Find the probability that the number is divisible by 3 and 7.

P(n divisible by 7 and n divisible by 3) = P(n divisible by 21).
There are floor (1000/21) = 47 positive integers in the range divisible by 21.
P(n divisible by 7 and n divisible by 3)= **47/1000**.

c) Find the probability that the number is divisible by 3 or 7.

P(n divisible by 7 or n divisible by 3) = P(n divisible by 7) + p(n divisible by 3) **–**
P(n divisible by 7 and n divisible by 3)
There are floor(1000/3) = 333 positive integers in the range divisible by 3.
Therefore P(n divisible by 3) = 333/1000.
From previous parts we get P(n divisible by 7) =142/1000 ,
P(n divisible by 7 and n divisible by 3) = 47/1000.
So P(n divisible by 7 or n divisible by 3)
= (333+142-47)/1000 = 428/1000 = **107/250**

d) Find the conditional probability that the number is divisible by 3 given that it's not divisible by 7.

P(n divisible by 3 | n not divisible by 7)
= P(n divisible by 3 and not divisible by 7)/ P(n not divisible by 7)
= (333-47)/ (1000-142) = 286/858 = 143/429=**1/3**.

Part B

3.

	Enrollment	**Failure**
Applied Television Viewing	0.5	0.3
Underwater Basket Weaving	0.25	0.1
Advanced Napping	0.25	0.4

P(student took television viewing | student failed the course) =
P(student took television viewing and student failed the course)/ P(student failed a course)
= (0.5 * 0.3) / (0.5*0.3 + 0.1*0.25 + 0.25*0.4) = 0.15/0.275 = 0.545454… =**6/11**.

4. 10 cards are randomly chosen from a standard deck of 52 cards.

This type of question can be done with a permutation oriented approach or a combination oriented approach. Both solutions are shown for part A and you can verify that they produce the same result. In parts B- D only the combination solution is shown. It is left as an exercise to use a permutation approach to solve B-D.

Numerical solutions are provided for those who actually performed the calculations. Note it is ok for the solutions to these questions to be left in terms of combinations or factorials.

a) What's the probability that exactly 5 of them are hearts?

Permutation oriented Approach

This type of approach is oriented towards the probability of selecting individual cards that make up the 10 cards.

P(exactly 5 hearts) = P(getting the first heart) * P(getting the second heart) *… * P(getting the 5^{th} heart) * P(getting the first non heart) * P(getting the second non heart) * …* P(getting the 5^{th} non heart) * C(10, 5).

Without the last factor, we have the probability of drawing 5 hearts then 5 non hearts. The last factor takes into account the fact there are actually C(10, 5) different combination of draws which can produce the hearts.

P(exactly 5 hearts) = **13/52 * 12/51 * 11/50 * 10/49 * 9/48 * 39/47 * 38/46 * 37/45 * 36/44 * 35/43 * C(10, 5)**.

Combination oriented Approach

This approach looks for the number of possible combinations of cards that satisfy the specified conditions out of the total number of possible combinations of 10 cards.

P(exactly 5 hearts) = (number of ways to choose 5 hearts * number of ways to choose 5 non hearts) / (number of ways to choose 10 cards)

P(exactly 5 hearts) = **C(13, 5) * C(39, 5) / C(52, 10)**.
Numerical solution = 0.046839.

b) What's the probability that 5 or more of them are hearts?

P(5 or more hearts) = 1- P(less than 5 hearts)
= 1-(P(0 hearts) + P(1 heart) + P(2 hearts) + P(3 hearts) + P(4 hearts))
= **1 – (C(13, 0) * C(39, 10) + C(13, 1) * C(39, 9) + C(13, 2) * C(39, 8) + C(13, 3) * C(39, 7) + C(13, 4) * C(39, 6)) / C(52, 10)** .
Numerical solution = 0.056814.

c) What's the probability that exactly 5 are hearts given that there are no spades among the 10?
P(exactly 5 hearts | no spades among the 10)
= P(exactly 5 hearts and no spades among the 10) / P(no spades among the 10)
= (C(13, 5) * C(26, 5) / C(52, 10)) / (C(39, 10) / C(52, 10))
= **C(13, 5) * C(26, 5) / C(39, 10)** .
Numerical solution= 0.133165.

d) What's the probability that there are no spades given that exactly 5 are hearts?

P(no spades among the 10 | exactly 5 hearts)
= P(no spades among the 10 and exactly 5 hearts) / P(exactly 5 hearts)
=(C(13, 5) * C(26, 5) / C(52, 10)) / (C(13, 5) * C(39, 5) / C(52, 10))
= **C(26, 5) / C(39, 5)**.
Numerical solution = 0.11425.

Alternately, we could have arrived at the same result in the following way:
P(no spades among the 10 | exactly 5 hearts)
= P(exactly 5 hearts | no spades among the 10) * P(no spades among the 10) / P(exactly 5 hearts) .

Part C

5(a). The expected payoff of the game played once can be calculated by multiplying the value of each card* probability of getting each card. In the first version of the game, this can be calculated as:

Payoff = -11 * 1/2 + 1/52 *(1+2+ …+ 12 +13) + 1/52 * 2*(1+2+…+ 12+13)
= -5.5 + 3* 91 /52 = **-0.25**

5(b) This game has the same rules as in the previous section except drawing an ace gives a payoff of $10.

Payoff
= -11 * 24/52 +4/52 * 10 + 1/52 * (2 +3+ …+12+ 13) +1/52 * 2* (2+3+…+ 12+13)
= -5.077+ 0.769+ 3*90/52
= +**0.88**

6. One strategy is the following:
<u>Algorithm</u>
-First, split the number of channels into two equal sets.
-Second, run through the first set and find the highest numerical value in this set. Call this *x*.

-Third, go through the channels in the second set and stop at the first channel that has a higher numerical ranking than *x*, or at the last channel if no channel with rating higher than *x* was found.

<u>Proof that this strategy gives a ¼ chance of stopping at the highest rated channel:</u>
The channels are distributed randomly and independently. There is a ½ chance that the second highest rated channel is in the first half of the channels. There is a ½ chance that the highest rated channel is in the second half of the channels. Therefore there is a ¼ chance that the second highest rated channel is in the first half and the highest rated channel is in the second half. *When this situation occurs*, the variable *x* will be the value of the second highest rated channel. There will be one channel in the second half that will have a value greater than *x* and this will be the highest rated channel.