

## CS 280 FALL 2001

### Homework 5 Grading Guide

#### Section 3.1

8) a) 1 point. Almost everybody got this one correct.

8) c and e) 2 points each. Errors:

Letting your Boolean variables take on non-boolean meanings. For example,  $p = \text{is insect}$ ,  $q = \text{is spider}$ . Those are not Boolean values.

Insects, spiders, food, tofu, and cheeseburgers should be sets.

16) a) 1 point – almost everyone had this correct.

16) b) 2 points – almost nobody got this correct.

First off, many people did not see that you needed to prove  $n^2$  is odd  $\rightarrow n$  is odd.

For those that saw this: you cannot let  $n^2 = (2k+1)^2$ . Neither can you let  $n^2 = 2k+1$  and then let  $k = 2m+1$ . This is the same error as letting  $n^2 = (2k+1)^2$ . Nor can you use any other such trickery to get  $n^2$  in the form of an odd number squared (except for a proof of course).

16) c) 2 points – almost everybody had this correct. Some people used their incorrect answer to part b as reasoning, but we didn't take off points for that. Just be careful next time!

30) 5 points. 1 point if you had all the cases, and 1 point for each case (if you had different cases, then the points were distributed accordingly). Most people had this question correct. There were a few proofs by example. If you had this, see the professor or go to office hours.

32) 5 points. 2 for the even case, 3 for the odd case. Strangely enough, the most common error was that people thought the floor and ceiling were brackets. That cost 3 points total.

Also, you cannot assume the conclusion and derive something true. It is not a valid method of proof. For example, in the  $n$  is even case, you cannot assume that  $\lfloor n/2 \rfloor \lceil n/2 \rceil = \lfloor n/4 \rfloor$  and using that derive that  $k^2 = k^2$ , where  $n = 2k$ .

#### Section 3.2

**Problem 3.2.4** (5 points) 1 point for showing the base case, 1 point for stating the induction hypothesis, 3 points for the induction step.

**Problem 3.2.28** (5 points) 2 points for showing the base case (this includes mentioning that  $n=2,3$  do not work), 1 point for stating the induction hypothesis, 2 points for the induction step. 1 point deducted if you assume  $2n+1 < n * n!$  or for other assumptions besides the IH in the IS.

**Problem 3.2.32** (5 points) 1 point for showing the base case (this includes mentioning that 5 and 15 cents cannot be formed), 1 point for stating the induction hypothesis, 3 points for the induction step. 1

point deducted if you mention removing a quarter and adding three dimes and not the other case (removing two dimes and adding a quarter) and vice-versa.

**Problem 3.2.40** (5 points) 1 point for showing the base case, 1 point for stating the induction hypothesis, 3 points for the induction step.

### Sections 3.3,3.4

Sec 3.3 (20 marks)

4) Each part is worth 1 mark, if fully correct.  
You get 1/2 if at least 2 out of 4 correct.

18) 5 marks for the 2 algorithms, 2.5 each.  
Base case is worth 1 mark, recursive formula is worth 1.5 marks.

Common mistake:

Wrong base case.

Max(a) should return a. A lot of people did not handle the case where we only need to find the max and min of one element.

22) 2 marks for each part, 6 marks in total.

Comments:

We accepted alternative answers such as  $p(n+1) = a_n x^{n+1} + p(n)$ ,  $p(n)$  denotes polynomial of degree  $n$ .

40) 5 marks. Credit was given to all steps of working shown even if the final answer was incorrect.

Sec 3.4 (10 marks)

6 & 22) 5 marks each

2 marks for base case

3 marks for recursive definition

Comments:

Although strictly speaking, the algorithm should compute remainder of  $x^n \bmod m$  where the remainder is less than  $m$ , we accepted answers such as  $(x \bmod m) * (\text{powermod}(x, n-1, m))$  which will not always give us an answer  $< m$ .

Some people did it by reducing  $x^n \bmod m$  by  $m$  each time. The problem with this is that the algorithm will take an extremely long time to run. Imagine reducing  $2^{1000}$  by 5 each time! Furthermore, there are also problems such as overflow, for example, the program cannot handle powers of  $x$  if  $n$  is too large.