CS 280 – Discrete Structures
HW 1 – Grading Guide

Section 1.1

1.1, 12)
Correct answer received 5 points.

If your answer was "yes, such a barber can exist", and gave no reason or completely incorrect reason, you received 0 points.
If your answer was "yes", but you gave some reason that made you look like you sort-of got the right idea, you received anything from 1 to 3 points.
Similarly, for the right answer ("no, cannot exist") but wrong reason, you received anything from 1 to 3 points.

Too many people got this question wrong. If a question like this appears on the exam, ask yourself, "What would a discrete math class want to test with this?" Think about possible contradictions. In most cases, there is a way to arrive at a contradiction without getting into complicated assumptions.

1.1, 14a)
Correct answer received 2.5 points.

If you said the problem was that both types of cannibals would answer "yes", you received 1.5 points (we assume that you understood the concept but just made an error while writing down).
If you said the problem was that one cannibal answers "yes" and the other answers "no," you received 0 to 0.5 points, depending on your reasoning.

1.1, 14b)
Correct answer received 2.5 points.

Ambiguous questions such as those for which we cannot be sure that the cannibal knows the answer, received partial credit, in most cases 1.5 points. An example of such an ambiguous question is "Is 1+1 = 3?" Yes it is a very simple question, but not as straightforwardly simple as "Are you a cannibal?" or "Is the sky blue?" Do not assume that the cannibal is smart. Ignorance is always a safer assumption.

Although an explanation was not necessary, and we did not cut any points for a missing explanation of how your question works, you should, in general include an explanation of your reasoning.

1.1, 28)
Correct answer for each part received 1 point.

Incorrect answer received 0 points, regardless of accompanying working or explanation of solution.
If you did not assume that x gets reset to 1 before each statement is executed, we did not cut any points as long as you got the following answers instead of those on the solutions: a) 2, b) 2, c) 3, d) 3, e) 3

Almost everyone got this question correct.

1.1, 42a)
Correct answer with correct reasoning received 5 points.

3 points for saying "John did it," and 2 points for your reasoning. Incorrect answers with some correct reasoning received some or all of the 2 points. Correct answers with incorrect or wrong reasoning received at least 3 points.

Part b was not asked and thus not graded.

Section 1.2

1.2, 18) When truth table is used for the proof, 0.5 point lost per wrong entry and 1 point lost per forgotten row.

1.2, 8a) 0.5 point lost per wrong entry and 1 point lost per forgotten row.

1.2, 8b) 0.5 point lost per wrong entry and 1 point lost per forgotten row.

1.2, 24) Any expression satisfying the relation is accepted.

1.2, 34a) 1 point lost for using wrong definitions such as p NOR q = ~ (p AND q). 0.5 point lost per wrong entry and 1 point lost per forgotten row.

1.2, 34b) 1 point lost for using wrong definitions such as p NOR q = ~ (p AND q). 0.5 point lost per wrong entry and 1 point lost per forgotten row.

1.2, 34c) Some people thought that it is sufficient to represent NOR in terms of OR and ~ for functional completeness of the NOR operator, which is not true. For these cases, 1 point is lost. Only restatements of the question and irrelevant answers got a 0.

If you used the conclusion from #29 implicitly, no points are lost. If you use the conclusion from (a) and (b) implicitly, 1 point is taken off.

1.2- Common mistakes

A surprising number of people had truth tables with missing rows. In general, a truth table for n variables should have 2^n rows.

1.2, 18)

1.2, 8a) Wrong entries in the truth table; mostly due to the wrong conception:

Wrong	Right
F->F=F	F->F=T
F->T=F	F->T=T

1.2, 8b) Wrong entries, forgotten rows in the truth table.

1.2, 24)

1.2, 34a) Repeated rows in the truth table such as:

These rows are unnecessary

P	~p	…..
F	T	…
F	T	…
T	F	..
T	F	..

Note: No points lost due to this mistake.

1.2, 34b)

1.2, 34c) Most of the mistakes in this question are in the concluding reasoning part. Most mistaken people thought that being able to express NOR in terms of OR and ~, which are functionally complete, is sufficient for NOR to be functionally complete. However, this is not true.

e.g: AND can be expressed by only using OR and ~ which are functionally complete. However, AND is not a functionally complete operator alone.

The reason should have been as below:

It is possible to express functionally complete operators, namely OR and ~, in terms of NOR, thus, NOR is also a functionally complete operator.

In future assignments, please be explicit about the premises of your conclusions. Saying that OR and NOT can be expressed with NOR is not enough. You should also explicitly state that since OR and NOT form a functionally complete collection, NOR is functionally complete.

Section 1.3

1.3.8:
For question 1.3.8(e) and 1.3.8(f), some students confused $\exists x \exists y \forall z$ with $\forall z \exists x \exists y$, and suggested that this means there is at least 1 student enrolled in ALL classes offered. This is incorrect. You should always take note that $\exists x \forall y$ and $\forall y \exists x$ are two completely different quantifications.

1.3.14:
For the questions here, alternative answers are correct as long as they are logically equivalent. Also, please include "$\forall x$" or "$\exists x$" if you are talking about the universe of discourse. It wouldn't make much sense to simply write ~C(Bob,x); does x refer to all possible x or a single x?

Next, you should also remember to put a relation operator between 2 propositions to generate a new proposition. For example, if you want to say "If John has an Internet connection, Sharon will also have an Internet connection," you write:
"I(John) → I(Sharon)," not "I(John)I(Sharon)."
The second proposition is not only wrong, but also very ambiguous; it could mean that both John and Sharon have Internet connections.

Lastly, the quantification "$\exists x \forall x$" contains redundancy; if you would like to say "for all x", use "$\forall x$", and if you would like to say "there exists an x/ there is at least one x," you write "$\exists x$," but never both at the same time.

1.3.14(e)
A number of students wrote $\forall x\ ((x \neq \text{Joseph}) \wedge C(\text{Sanjay}, x))$ as an answer. This is logically different from the correct answer $\forall x\ ((x \neq \text{Joseph}) \rightarrow C(\text{Sanjay}, x))$. Furthermore, it does not make sense to say that "For all x, x is not Joseph" because there must be at least one x in the universe of discourse whose name is Joseph."

Also, it is incorrect to suggest $\forall x\ (C(\text{Sanjay}, x) \wedge C(\text{Sanjay}, \text{Joseph}))$, because if x = Joseph, the statement will be self-contradictory.

1.3.14(h), (i)
The correct answer must definitely suggest that there exists *one and only one* student who has/ does not have Internet connection.

Thus, for 14(h), answers such as $\exists x \forall y\ ((y \neq x) \rightarrow \sim I(y))$ are wrong, because they do not suggest that there is anyone who has an Internet connection.
Answers such as: $\exists x \forall y\ (I(x) \wedge (I(y) \rightarrow (x=y)))$ $\exists x \forall y\ ((y \neq x) \leftrightarrow \sim I(y))$ are also correct, since they suggest the same thing (even though they are both logically different from the answer given)
Also, it is acceptable to place the $(x \neq y)$ condition together with I(x):
$\exists x \forall y\ ((I(x) \wedge (y \neq x)) \rightarrow \sim I(y))$

It is incorrect to suggest $\exists x \forall y\ (I(x) \wedge \sim I(y) \wedge (x \neq y))$ too because if $x = y$, the statement is self contradictory and by definition, always false. (Refer to the explanation given in 1.3.14(e))

The same rules and conditions apply for 1.3.14(i), since the two statements are almost equivalent; simply replace $I(x)$ by $\sim I(x)$ and $I(y)$ by $\sim I(y)$ in 14(h) and you will get 14(i).

1.3.22:
Most students got 1.3.22(h) incorrect; once again, it is important to highlight the fact that $\exists x \forall y$ and $\forall y \exists x$ are two completely different quantifications, thus to suggest that if $x = 1/y$ then $xy = 1$ does not fit the question because x cannot be a variable in the question.
$\exists x \forall y \neq 0\ (xy = 1)$ means that there must exist *one single value* for x such that for all y, xy will be equal to 1. This is clearly not the case, and therefore, the statement is false.

1.3.34:
For 1.3.34(a), answers such as:
$\forall x \sim(P(x) \wedge S(x))$ are logically equivalent to $\forall x\ (P(x) \rightarrow \sim S(x))$, and are therefore acceptable.
Similarly, for 1.3.34(b),
$\forall x \sim(R(x) \wedge \sim S(x))$ is logically equivalent to $\forall x\ (R(x) \rightarrow S(x))$, so both answers are acceptable as well.