CS 280 Solutions – Assignment 6

Section 4.1

22 (a) Number of strings of 4 decimal digits that do not contain the same digit twice
= 10 x 9 x 8 x 7
= 5 040

(b) Number of strings of 4 decimal digits that end with an even digit
= (# possibilities for first 3 digits) x (# possibilities for last digit)
= 10 x 10 x 10 x 5
= 5 000

(c) Number of strings of 4 decimal digits that have exactly 3 digits that are 9s
= (Number of possible ways to arrange the 3 9s) x (# possibilities for last digit)
= C (4, 3) x 9
= 4 x 9
= 36

38 (a) Number of ways to arrange the row if the bride is in the picture
= (Number of ways to arrange bride) x (# possibilities for remaining 5 people)
= 6 x (9 x 8 x 7 x 6 x 5)
= 90 720

(b) Number of ways to arrange the row if both bride and groom (B&G) are in picture
= (Number of ways to arrange B&G) x (# possibilities for remaining 4 people)
= P (6, 2) x (8 x 7 x 6 x 5)
= 50 400

Alternatively,
Number of ways to arrange the row if both bride and groom (B&G) are in picture
= (# ways to select 4 remaining people) x (# ways to arrange 6 people)
= C (8, 4) x (6!)
= 70 x 720
= 50 400

(c) Number of ways to arrange the row if exactly one of the bride and the groom is in the picture
= (# ways to arrange bride) x (# possibilities for remaining 5 people) +
(# ways to arrange groom) x (# possibilities for remaining 5 people)
= 6 x (8 x 7 x 6 x 5 x 4) + 6 x (8 x 7 x 6 x 5 x 4)
= 80 640

Section 4.2

10 (x_i, y_i) can take on one of these four categories:
(O,O), (O,E), (E,O), (E,E)
in which O represents an "odd integer" and E represents an "even integer"

Since there are 5 distinct points (x_1, y_1) ... (x_5, y_5),
it follows from the pigeonhole principle that at least 2 of the points fall into the same category. The midpoint of these points would be $(\frac{1}{2} (x_i + x_j, y_i + y_j))$.

However, since x_i and x_j are both odd, or both even, their sum will be an even number. Thus $\frac{1}{2} (x_i + x_j)$ will be an integer coordinate, since halving an even integer results in another integer. Similarly, since y_i and y_j are both odd, or both even, $\frac{1}{2} (y_i + y_j)$ will be an integer coordinate as well.

Thus, the midpoint of these 2 points in the same category will have integer coordinates. Since the pigeonhole principle has already established the existence of these 2 points, there must also exist a midpoint between 2 vertices with integer coordinates, as previously proved.

26 Assuming that wage earners do not earn zero income, the amount of money, M, earned by a wage earner falls into this range:

$0.01 ≤ M ≤ $999 999.99

There are 99 999 999 distinct income levels for M, to the penny.

There are 100 000 000 distinct wage earners in the United States.

Since the number of wage earners exceeds the number of distinct income levels, by pigeonhole principle, there exist 2 people who are in the same income level, i.e, they earned exactly the same amount of money, to the penny, last year.

Section 4.3

16 (a) Bit strings of length 10 with exactly 3 zeros
= C (10, 3)
= 120

(b) Bit strings of length 10 with same number of zeros and ones
= C (10, 5)
= 252

(c) Bit strings of length 10 with at least 7 ones
= C (10, 7) + C (10, 8) + C (10, 9) + C (10, 10)
= 176

(d) Bit strings of length 10 with at least 3 ones
= C (10, 3) + C (10, 4) + C (10, 5) + C (10, 6) + C (10, 7) + C (10, 8) + C (10, 9) + C (10, 10)
= 968

Alternatively,
Bit strings of length 10 with at least 3 ones
= 1024 – bit strings of length 10 with at most 2 ones
= 1024 – C (10,0) – C(10,1) – C(10,2)
= 968

18 (a) Number of ways to choose 10 players to take the field
= C (13, 10)
= 286

(b) Number of ways to assign 10 positions
= P (13, 10)
= 1 037 836 800

(c) Number of ways to choose 10 players (at least 1 woman)
= Total number of ways to choose 1 woman 9 men, 2 women, 8 men and 3 women, 7 men
= C (3, 1) x C (10, 9) + C (3, 2) x C (10, 8) + C (3, 3) x C (10, 7)
= 3 x 10 + 3 x 45 + 1 x 120
= 285

28. Suppose "011" is a unit
Then, number of bit strings that contain exactly five 0s and fourteen 1s if every 0 must be followed by two 1s
= Number of arrangement of 5 "011"s and 4 "1"s
= C (9,5)
= 126

40. Coefficient of x^8y^9 = C (17, 9) x $(3)^8(2)^9$
= 81 662 929 920