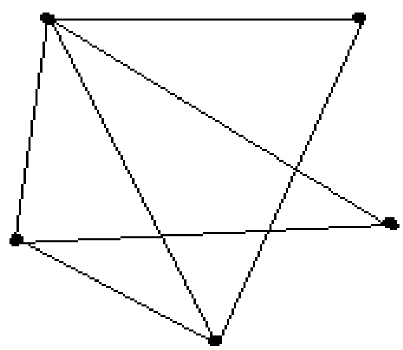
Assignment 10 Solutions

Section 7.2:

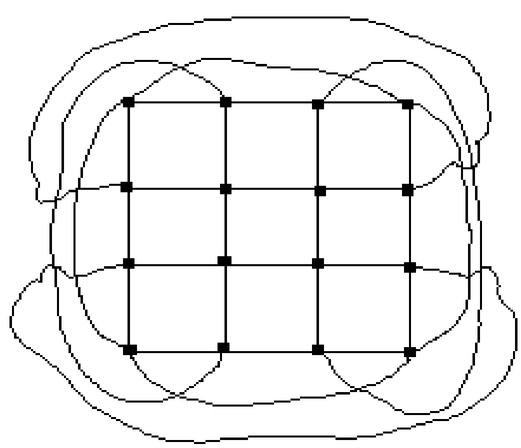
6.Construct the graph *G=(V,E)* as follows. Let each person at the party be a vertex. If person *a* and person *b* shake hands, let there be an edge between vertex *a* and vertex *b*. Then it is clear that the number of people a person has shaken hands with is the degree of the corresponding vertex. Then by "The Handshaking Theorem", Theorem 1, the sum over the degree of the vertices is even, i.e. the sum, over the set of people at the party, of the number of people a person has shaken hands with, is even.

20.Applying Theorem 1, we see that the number of edges *e*=(4+3+3+2+2)/2=7. An



example of such a graph is

44.A graph of the variant of the mesh network for 16 processors:



Section 7.3:

30. By definition, an incidence matrix ***M*** has m(i,j)=1 when edge e(j) is incident with v(i), and 0 otherwise. Thus the sum of entries in a row of ***M*** is the total number of edges incident with the vertex corresponding to that row, i.e. It is the degree of that vertex.

44. We shall show that the given graphs are not isomorphic. To do this, assume that there is an isomorphism f such that $f(u_i)=v_{k(i)}$ for i=1,...,8. Note that $k(i)$ is an integer between 1 and 8, dependent on i, and $k(i)=k(j)$ if and only if $i=j$. By symmetry of the second graph, we may assume that $k(1)=1$, i.e. $f(u_1)=v_1$. Then since f is an isomorphism, we must have that $\{f(u_3), f(u_7)\}=\{v_4, v_6\}$. Assume first that $f(u_3)=v_4$ and $f(u_7)=v_6$. Then $f(u_3)=v_4$ implies that $f(u_5)=v_7$. But $f(u_7)=v_6$ implies that $f(u_5)=v_3$. This is a contradiction. Now assume that $f(u_3)=v_6$ and $f(u_7)=v_4$. Then $f(u_3)=v_6$ implies that $f(u_5)=v_3$, but $f(u_7)=v_4$ implies that $f(u_5)=v_7$. Once again we have a contradiction and since we have covered all the possible cases, an isomorphism cannot exist and the two graphs are not isomorphic.

68.

a. Each edge in a graph is specified by a pair of vertices. Thus each edge is represented twice in an adjacency list, once for each vertex. Thus the total storage required for an adjacency list is $2e$ or $O(e)$ or $O(v^2)$.

b. An adjacency matrix is a vxv matrix and so the storage required is v^2 or $O(v^2)$.

c. An incidence matrix is a vxe matrix and so the storage required is ve or $O(ve)$ or $O(v^3)$.