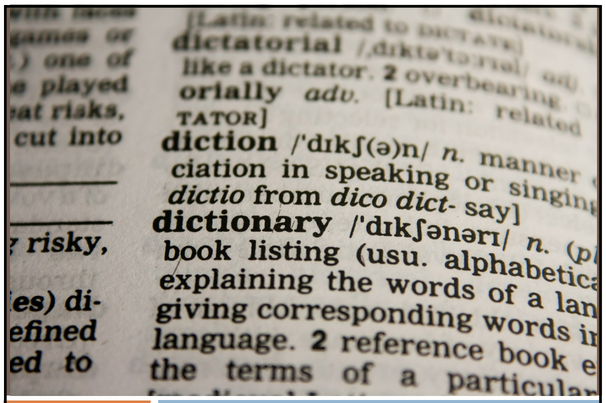


HASHING

CS2110

Announcements

2

- Submit Prelim 2 conflicts by Thursday night
- A6 is due Nov 7 (*tomorrow!*)

Ideal Data Structure

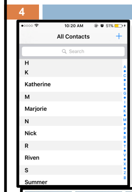
3

Data Structure	add(val x)	get(int i)	contains(val x)
ArrayList 2 1 3 0	$O(n)$	$O(1)$	$O(n)$
LinkedList ②▶①▶③▶⓪	$O(1)$	$O(n)$	$O(n)$
Goal:	$O(1)$	$O(1)$	$O(1)$

AKA add, lookup, search

Mystery Data Structure in Your Life

4



What do these data structures have in common?

New Data Structure: Hash Set

5

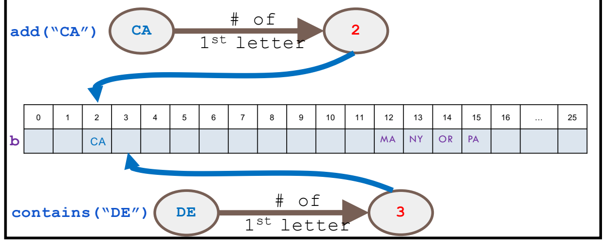
Data Structure	add(val x)	get(int i)	contains(val x)
ArrayList 2 1 3 0	$O(n)$	$O(1)$	$O(n)$
LinkedList ②▶①▶③▶⓪	$O(1)$	$O(n)$	$O(n)$
HashSet 0 1 2 3 / 3 _ 1 2	$O(1)$	$O(1)$	$O(1)$

Expected time
Worst-case: $O(n)$

AKA add, lookup, search

Intuition behind a Hash Set

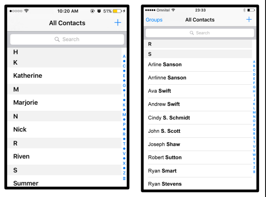
Idea: finding an element in an array takes constant time when you know which index it is stored in.
So… let's place elements in the array based on their starting letter! (A=0, B=1, …)



What could possibly go wrong?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	…	25
b	AL		CA	DE		FL	GA						MA	NY	OR	PA			

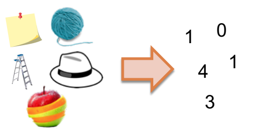
- Some buckets get used quite a bit!
  - called **Collisions**
- Not all buckets get used



H
K
Katherine
M
Marjorie
N
Nick
R
Riven
S
Summer

R
S
Arline **Sanson**
Arrlinne **Sanson**
Ava **Swift**
Andrew **Swift**
Cindy **S. Schmidt**
John **S. Scott**
Joseph **Shaw**
Robert **Sutton**
Ryan **Smart**
Ryan **Stevens**

Hash Functions



- Requirements:
  1) deterministic
  2) return a number*
- Properties of a good hash:
  1) fast
  2) collision-resistant
  3) evenly distributed
  4) hard to invert

* the number is either in [0..n-1] where n is the size of the Hash Set, or you compute the hash and then % n, constraining it to be in [0…n-1]

Example: hashCode()

- Method defined in java.lang.Object
- Default implementation: uses memory address of object
  - If you override equals, you must override hashCode!!!
- String overrides hashCode:

$$\text{s.hashCode()} := s[0] * 31^{n-1} + s[1] * 31^{n-2} + \ldots + s[n-1]$$

Do we like this hashCode?

Can we have perfect hash functions?

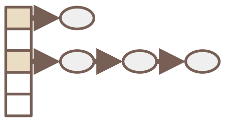
- Perfect hash functions map each value to a different index in the hash table

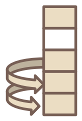
- Impossible in practice
- ● Don't know size of the array
- ● Number of possible values far far exceeds the array size
- ● No point in a perfect hash function if it takes too much time to compute

Collision Resolution

Two ways of handling collisions:

1. Chaining
2. Open Addressing

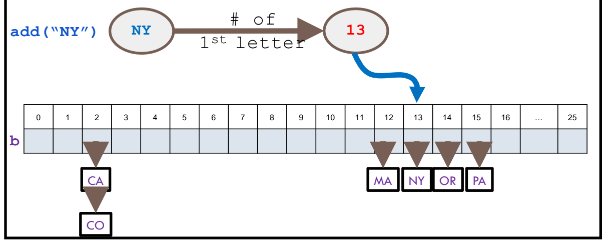




Chaining (1)

```
add("NY")
```

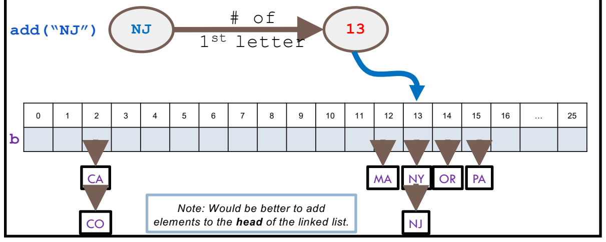
Each bucket is the beginning of a Linked List



Chaining (2)

add("NY")
add("NJ")

Each bucket is the beginning of a Linked List

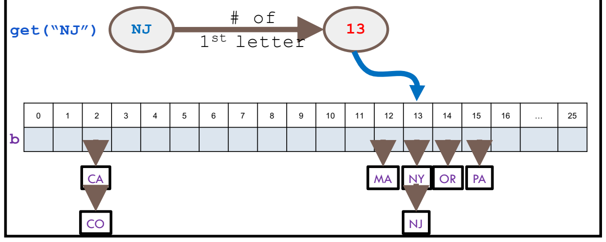


Note: Would be better to add elements to the ***head*** *of the linked list.*

Chaining (3)

add("NY")
add("NJ")
get("NJ")

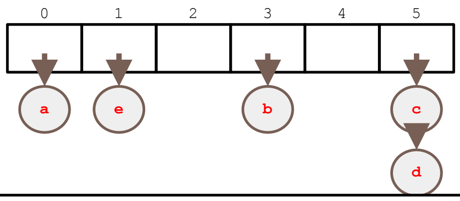
Each bucket is the beginning of a Linked List



Chaining in Action

Insert the following elements (in order) into an array of size 6:
if `code > n`, use `(code % n_buckets)`

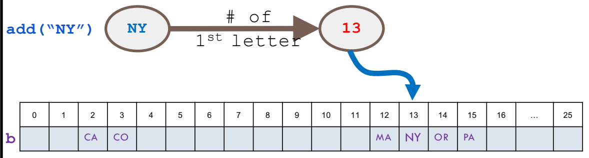
element	a	b	c	d	e
hashCode	0	9	17	11	19



Open Addressing (1)

add("NY")

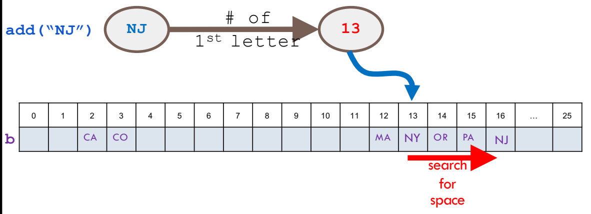
Probing: Find another available space



Open Addressing (2)

add("NY")
add("NJ")

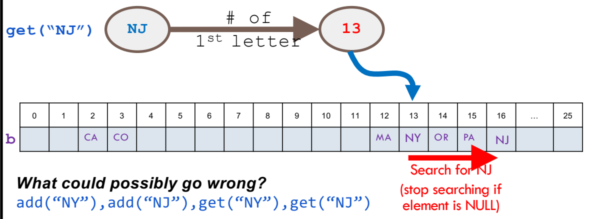
Probing: Find another available space



Open Addressing (3)

add("NY")
add("NJ")
...
get("NJ")

Probing: Find another available space

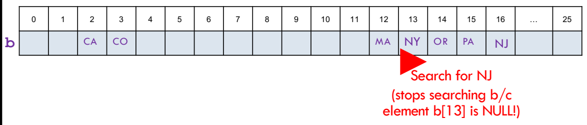


What could possibly go wrong?
`add("NY"),add("NJ"),get("NY"),get("NJ")`

Deletion Problem w/Open Addressing

```
add("NY")
add("NJ")
get("NY")
get("NJ")
```

Probing: Find another available space

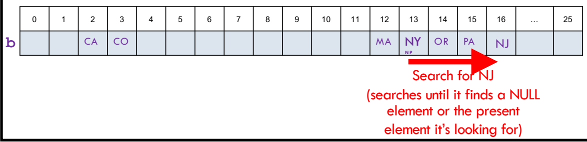


Deletion Solution for Open Addressing

```
add("NY")
add("NJ")
get("NY")
get("NJ")
```

Probing: Find another available space

Need to mark element as "not present"
Indicates to search that it should keep looking


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
...
25
b
CA
CO
MA
NY
NP
OR
PA
NJ
Search for NJ
(searches until it finds a NULL element or the present element it's looking for)

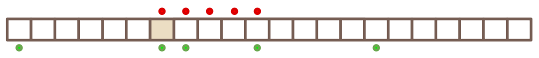
Different probing strategies

When a collision occurs, how do we search for an empty space?

clustering: problem where nearby hashes have very similar probe sequence so we get more collisions

linear probing: search the array in order: **i**, **i+1**, **i+2**, **i+3** . . .

quadratic probing: search the array in nonlinear sequence: **i**, **i+1²**, **i+2²**, **i+3²** . . .

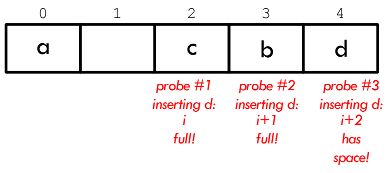


In order to have access to every bucket, important for size to be a prime number when using quadratic probing.

Linear Probing in Action

Insert the following elements (in order) into an array of size 5:

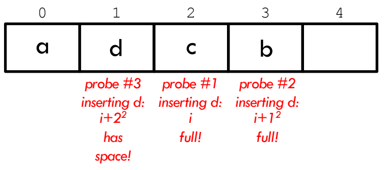
element	a	b	c	d
hashCode	0	8	17	12


0
1
2
3
4
a
c
b
d
probe #1 inserting d: i full!
probe #2 inserting d: i+1 full!
probe #3 inserting d: i+2 has space!

Quadratic Probing in Action

Insert the following elements (in order) into an array of size 5:

element	a	b	c	d
hashCode	0	8	17	12


0
1
2
3
4
a
d
c
b
probe #3 inserting d: i+2² has space!
probe #1 inserting d: i full!
probe #2 inserting d: i+1² full!

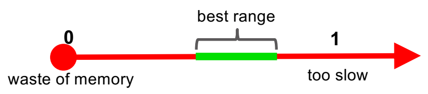
Load Factor

Load factor

$$\lambda = \frac{\textit{\# of entries}}{\textit{length of array}}$$

If load factor = ½, expected # of probes is 2.
What happens when the array becomes too full?
i.e. load factor gets a lot bigger than ½?

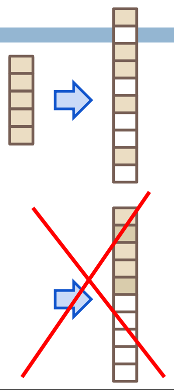
no longer expected constant time operations



Resizing

Solution: ***Dynamic resizing***

- double the size*
- reinsert / rehash all elements to new array
- Why not simply copy into first half?



*if using quadratic probing, use a prime >2n

Collision Resolution Summary

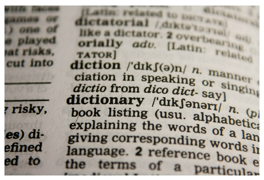
Chaining

- store entries in separate chains (linked lists)
- can have higher load factor/degrades gracefully as load factor increases

Open Addressing

- store all entries in table
- use linear or quadratic probing to place items
- uses less memory
- clustering can be a problem — need to be more careful with choice of hash function

Application: Hash Map



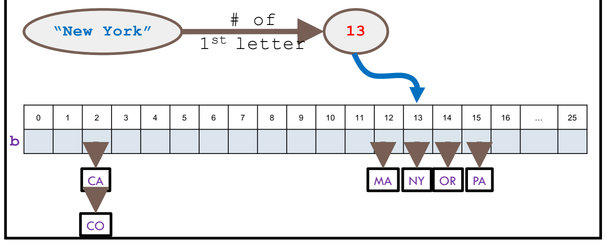
```
Map<K,V>{

    void put(K key, V value);

    void update(K key, V value);

    V get(K key);

    V remove(K key);

}
```

- Use the **key** for lookups
- Store the **value**

Example: **key** is the word, **value** is its definition

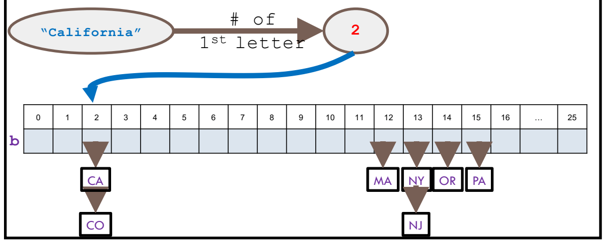
Hash Map (1)

```
put("New York","NY")
```



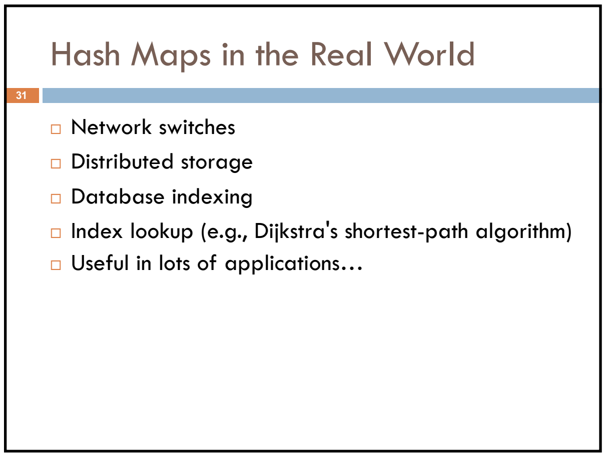
Hash Map (2)

```
put("New York","NY")

get("California")
```



HashMap in Java

- Computes hash using key.hashCode()
  - No duplicate keys
- Uses chaining to handle collisions
- Default load factor is .75
- Java 8 attempts to mitigate worst-case performance by switching to a BST-based chaining!



Hash Maps in the Real World

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- Network switches
- Distributed storage
- Database indexing
- Index lookup (e.g., Dijkstra's shortest-path algorithm)
- Useful in lots of applications…