HEAPS & PRIORITY QUEUES

Lecture 13
CS2110 Spring 2018

Announcements

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- A4 goes out today!
- Prelim 1:
  - regrades are open
  - a few rubrics have changed
- No Recitations next week (Fall Break Mon & Tue)
- *We'll* spend Fall Break taking care of loose ends

Abstract vs **concrete** data structures

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- Abstract data structures are **interfaces**
  - specify only **interface** (method names and specs)
  - not **implementation** (method bodies, fields, …)
  - Have multiple possible implementations
- Concrete data structures are **classes**
  - These **are** the multiple possible implementations

Abstract data structures (the interfaces)

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Interface	definition
List	an ordered collection (aka sequence)
Set	collection that contains no duplicate elements
Map	maps keys to values, no duplicate keys
Stack	a last-in-first-out (LIFO) stack of objects
Queue	collection for holding elements prior to processing
Priority Queue	*later this lecture!*

These definitions specify an interface for the user.
How you implement them is up to you!

Abstract data structures made concrete

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Interface	Class (implementation)
List	ArrayList, LinkedList ⇦
Set	HashSet, TreeSet
Map	HashMap, TreeMap
Stack	can be done with a LinkedList
Queue	can be done with a LinkedList

2 classes that both implement List

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- **List** is the **interface** ("abstract data type")
  - has methods: add, get, remove, …
- These **2 classes** implement List ("concrete data types"):

Class:	ArrayList	LinkedList
Backing storage:	array	chained nodes
add(i, val)	O(n)	O(n)
add(0, val)	O(n)	O(1)
add(n, val)	O(1)	O(1)
get(i)	O(1)	O(n)
get(0)	O(1)	O(1)
get(n)	O(1)	O(1)

Priority Queue

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Unbounded queue with ordered elements

→ data items are `Comparable` (ties broken arbitrarily)

Priority order: **smaller** (determined by `compareTo()`) have **higher priority**

`remove()`: remove and return element with highest priority

Many uses of priority queues

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Surface simplification [Garland and Heckbert 1997]

- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest M values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling
- College: prioritizing assignments for multiple classes.

java.util.PriorityQueue<E>

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```
interface PriorityQueue<E> {
 boolean add(E e); //insert e.
 E poll(); //remove/return min elem.
 E peek() //return min elem.
 void clear() //remove all elems.
 boolean contains(E e);
 boolean remove(E e);
 int size();
 Iterator<E> iterator();
}
```

Priority queues can be maintained as:

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A list
- `add()` put new element at front – O(1)
- `poll()` must search the list – O(n)
- `peek()` must search the list – O(n)

An ordered list
- `add()` must search the list – O(n)
- `poll()` min element at front – O(1)
- `peek()` O(1)

A red-black tree (*we'll cover later!*)
- `add()` must search the tree & rebalance – O(log n)
- `poll()` must search the tree & rebalance – O(log n)
- `peek()` O(log n)

Can we do better?

A Heap..

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Is a binary tree satisfying 2 properties

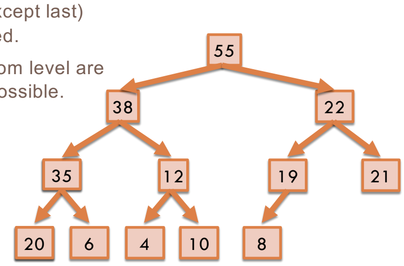
1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.

Do not confuse with heap memory, where a process dynamically allocates space–different usage of the word heap.

Completeness Property

Every level (except last) completely filled.

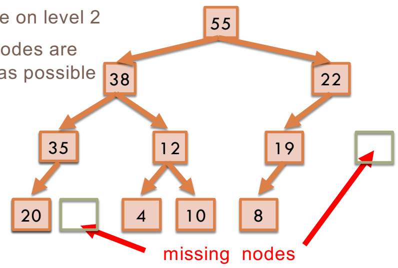
Nodes on bottom level are as far left as possible.



Completeness Property

Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible



A Heap..

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Is a binary tree satisfying 2 properties

1) **Completeness.** Every level of the tree (except last) is completely filled, and on last level nodes are as far left as possible.
2) **Heap Order Invariant.** *"max on top"*

   Max-Heap: every element in tree is <= its parent

   Min-Heap: every element in tree is >= its parent *"min on top"*

Order Property (max-heap)

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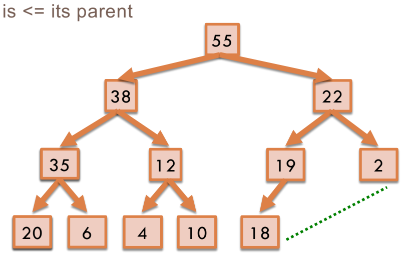
Every element is <= its parent



Note: Bigger elements **can** be deeper in the tree!

Heap Quiz #1

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A Heap..

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Is a binary tree satisfying 2 properties

1) **Completeness.** Every level of the tree (except last) is completely filled. All holes in last level are all the way to the right.
2) **Heap Order Invariant.**

   Max-Heap: every element in tree is <= its parent

Implements 3 key methods:

1) **add**(e): add a new element to the heap
2) **poll**(): delete the max element and returns it
3) **peek**(): return the max element

Heap: add(e)

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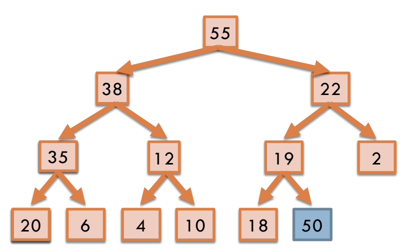

1. Put in the new element in a new node (leftmost empty leaf)

Heap: add(e)

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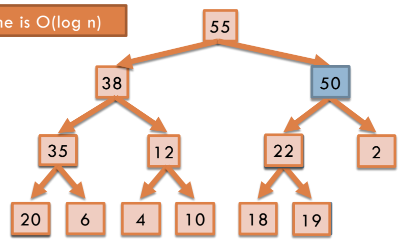
Time is O(log n)



1. Put in the new element in a new node (leftmost empty leaf)
2. Bubble new element up while greater than parent

Heap: poll()

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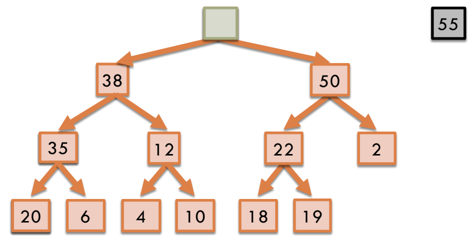

1. Save root element in a local variable

Heap: poll()

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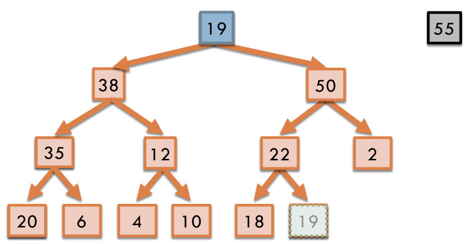

1. Save root element in a local variable
2. Assign last value to root, delete last node.

Heap: poll()

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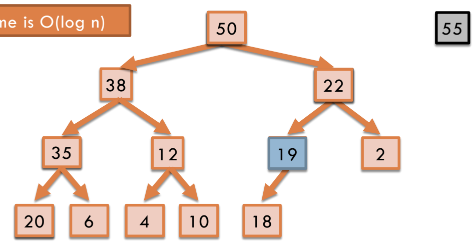
Time is O(log n)



1. Save root element in a local variable
2. Assign last value to root, delete last node.
3. While less than a child, switch with bigger child (bubble down)

Heap: peek()

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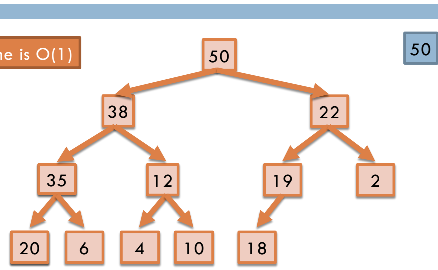
Time is O(1)



1. Return root value

Implementing Heaps

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```
public class HeapNode<E> {
  private E value;
  private HeapNode left;
  private HeapNode right;
  ...
}
```

Implementing Heaps

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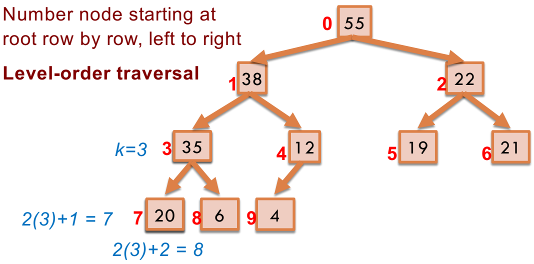
```
public class Heap<E> {
  private E[] heap;
  ...
}
```

Numbering the nodes

Number node starting at root row by row, left to right

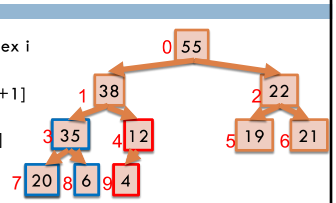
Level-order traversal

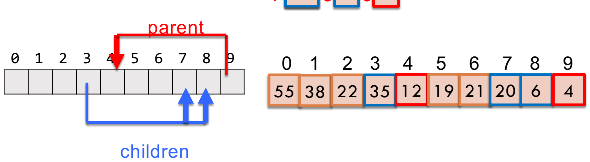


Children of node k are nodes 2k+1 and 2k+2
Parent of node k is node (k-1)/2

Storing a heap in an array

- Store node number i in index i of array b
- Children of b[k] are b[2k +1] and b[2k +2]
- Parent of b[k] is b[(k-1)/2]





add() (assuming there is space)

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```
/** An instance of a heap */
class Heap<E> {
  E[] b= new E[50];  // heap is b[0..n-1]
  int n= 0;          // heap invariant is true

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // given on next slide
  }
}
```

add(). Remember, heap is in b[0..n-1]

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```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
    int p= (k-1)/2;
    // inv: p is parent of k and every elmnt
    // except perhaps k is <= its parent
    while (k > 0  &&  b[k].compareTo(b[p]) > 0) {
        swap(b[k], b[p]);
        k= p;
        p= (k-1)/2;
    }
}
```

poll(). Remember, heap is in b[0..n-1]

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```
/** Remove and return the largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v=  b[0];   // largest value at root.
    n= n – 1;     // move last
    b[0]= b[n];   // element to root
    bubbleDown(0);
    return v;
}
```

poll()

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```
/** Tree has n node.
 *  Return index of bigger child of node k
    (2k+2 if k >= n) */
public int biggerChild(int k, int n) {
    int c= 2*k + 2;      // k's right child
    if (c >= n || b[c-1] > b[c])
       c= c-1;
    return c;
}
```

poll()

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```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k= 0;
    int c= biggerChild(k, n);
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s biggest child
   while ( c < n &&  b[k] < b[c]) {
       swap(b[k], b[c]);
       k= c;
       c= biggerChild(k, n);
   }
}
```

peek(). Remember, heap is in b[0..n-1]

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```
/** Return largest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    return b[0];   // largest value at root.
}
```

Heap Quiz #2

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HeapSort

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0	1	2	3	4
55	4	12	6	14

Goal: sort this array in place

HeapSort

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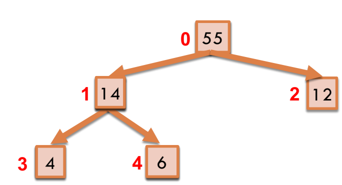
// Make b[0..n-1] into a max-heap (in place)

0	1	2	3	4
55	14	12	4	6



HeapSort

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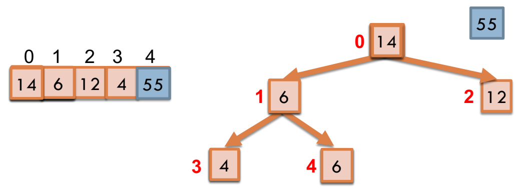
// Make b[0..n-1] into a max-heap (in place)
// inv: b[0..k] is a heap, b[0..k] <= b[k+1..], b[k+1..] is sorted
for (k= n-1; k > 0; k= k-1) {
b[k]= poll – *i.e.*, take max element out of heap.
}



HeapSort

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// Make b[0..n-1] into a max-heap (in place)
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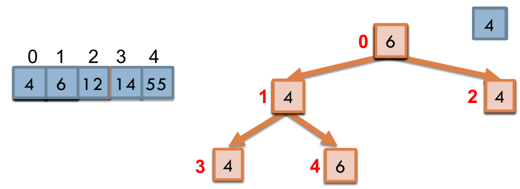
for (k= n-1; k > 0; k= k-1) {
b[k]= poll – *i.e.*, take max element out of heap.
}



Priority queues as heaps

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- A *heap* can be used to implement priority queues
  - Note: need a min-heap instead of a max-heap
- Gives better complexity than either ordered or unordered list implementation:
  - **add():** O(log n) (n is the size of the heap)
  - **poll():** O(log n)
  - **peek():** O(1)

java.util.PriorityQueue<E>

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```
interface PriorityQueue<E> {              TIME*
 boolean add(E e); //insert e.            log
 void clear(); //remove all elems.
 E peek(); //return min elem.             constant
 E poll(); //remove/return min elem.      log
 boolean contains(E e);                   linear
 boolean remove(E e);                     linear
 int size();                              constant
 Iterator<E> iterator();
}                        *IF implemented with a heap!
```

What if priority is independent from the value?

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Separate priority from value and do this:

add(e, p); //add element e with priority p (a double)

THIS IS EASY!

Be able to change priority

change(e, p); //change priority of e to p

THIS IS HARD!

Big question: How do we find e in the heap?
Searching heap takes time proportional to its size! No good!
Once found, change priority and bubble up or down. OKAY

Assignment A4: implement this heap! Use a second data structure to make change-priority expected log n time