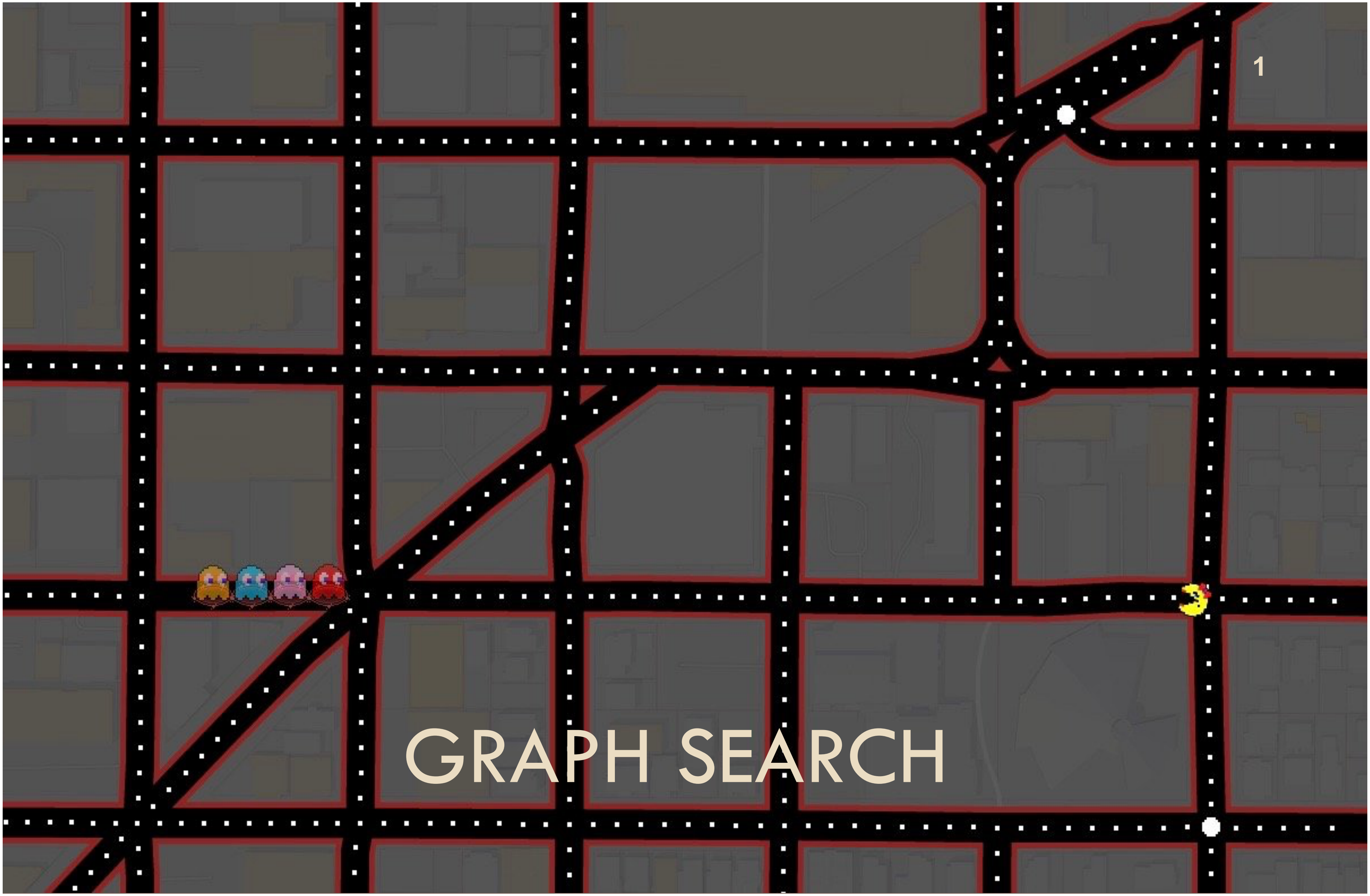


GRAPH SEARCH

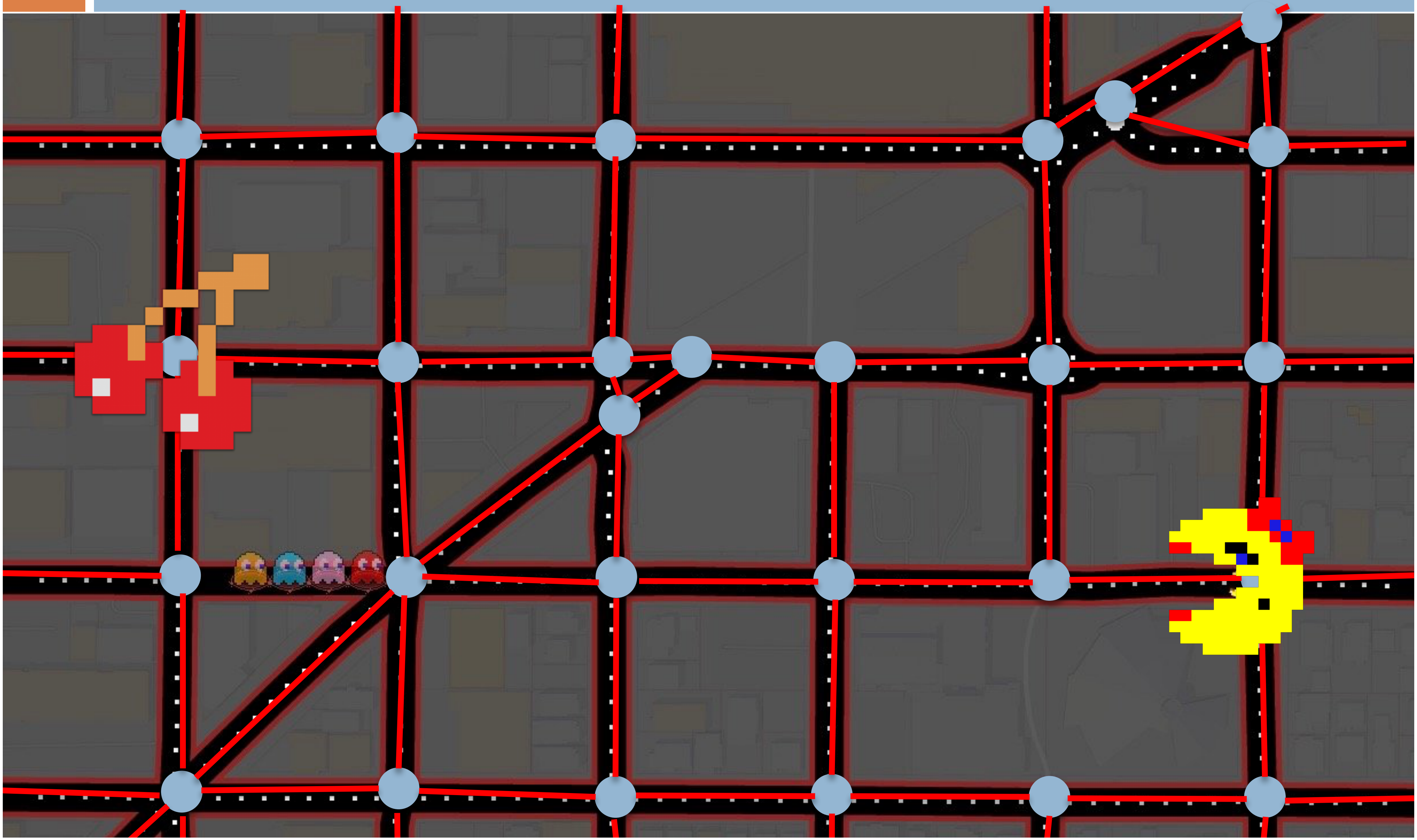
Lecture 17

CS 2110 Fall 2017

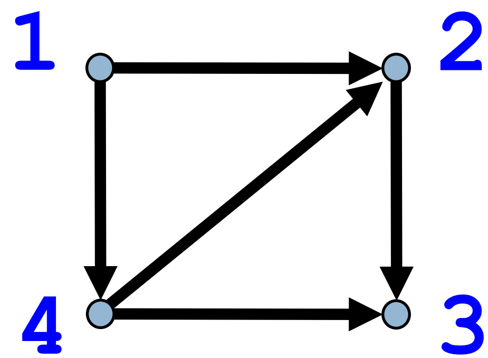
Announcements

- For the next lecture, you **MUST** watch the tutorial on the shortest path algorithm beforehand: http://www.cs.cornell.edu/courses/cs2110/2017fa/online/shortestPath/shortestPath.html
- Thursday's lecture **will assume** that you understand it. Watch the tutorial once or twice and execute the algorithm on a small graph.

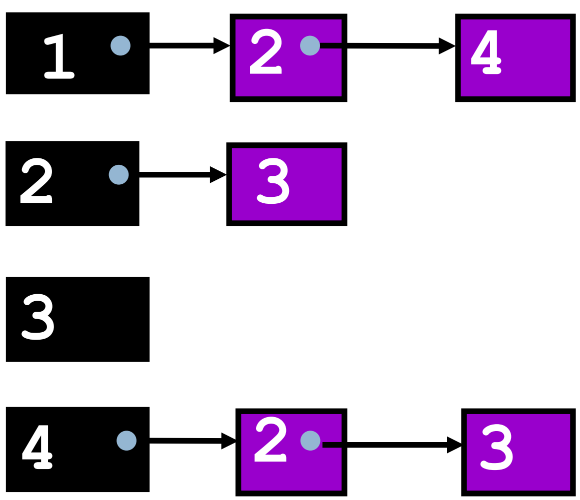
Graphs



Representing Graphs



Adjacency List



Adjacency Matrix

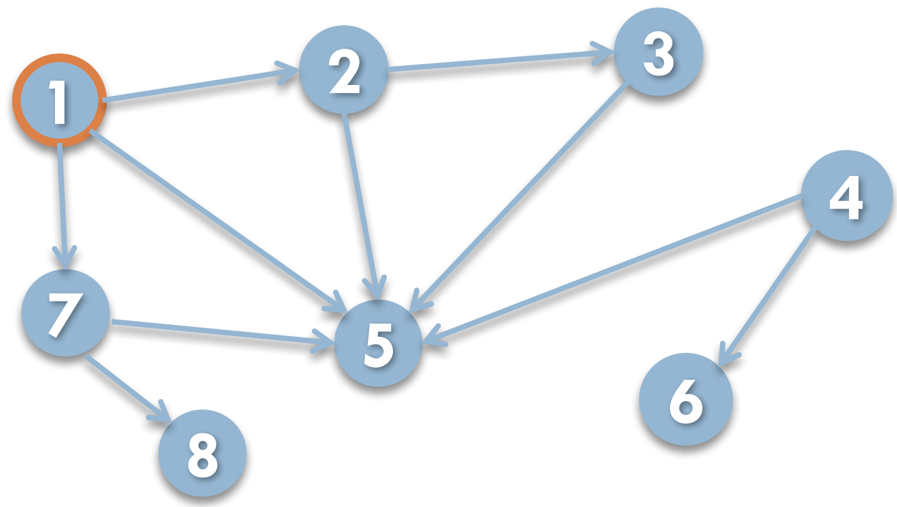
	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	0	0	0	0
4	0	1	1	0

Graph Algorithms

- Search
  - Depth-first search
  - Breadth-first search
- Shortest paths
  - Dijkstra's algorithm
- Spanning trees
    - Algorithms based on properties
    - Minimum spanning trees
  - Prim's algorithm
  - Kruskal's algorithm

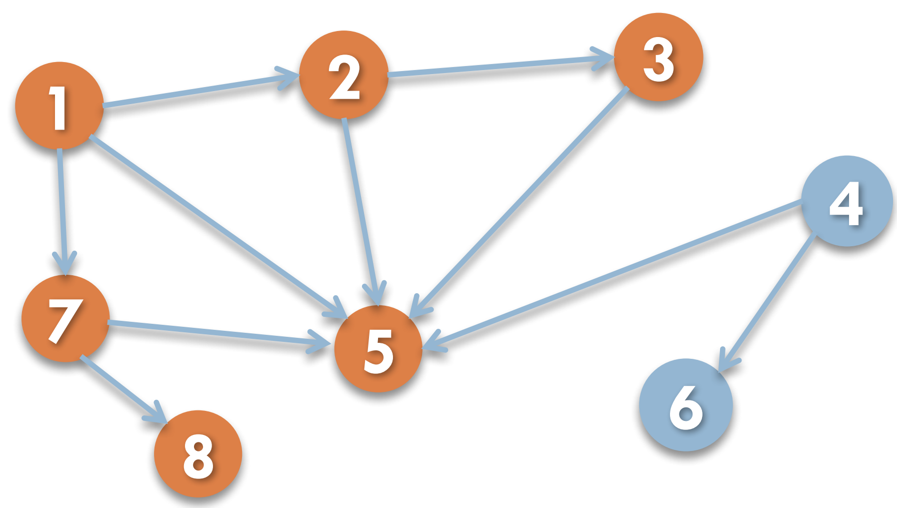
Search on Graphs

- Given a graph (V, E) and a vertex $u \in V$
- We want to "visit" each node that is reachable from u



There are many paths to some nodes.

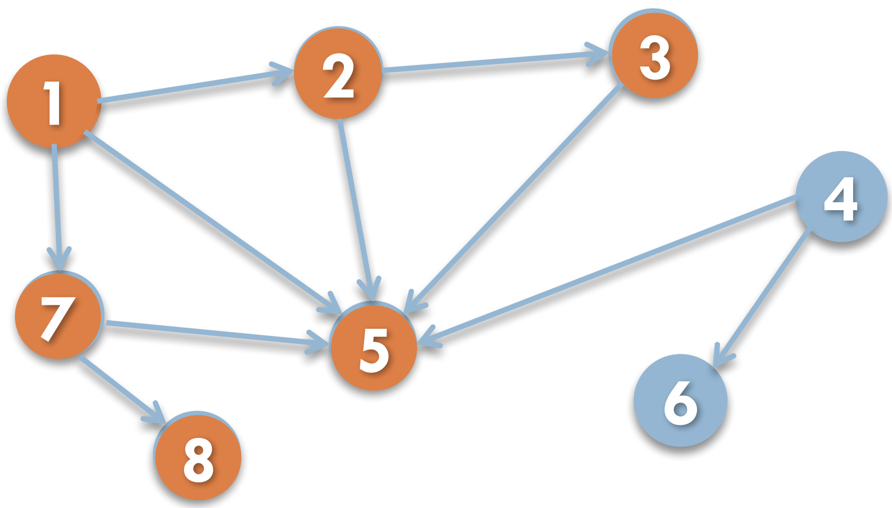
How do we visit all nodes efficiently, without doing extra work?



Depth-First Search

Intuition: Recursively visit all vertices that are reachable along unvisited paths.

```
/** Visit all nodes reachable
on unvisited paths from u.
Precondition: u is unvisited.
*/
public static void dfs(int u)
{
    visited[u] = true;
    for all edges (u,v):
        if(!visited[v]):
            dfs(v);
}
```



dfs(1) visits the nodes in this order: 1, 2, 3, 5, 7, 8

Depth-First Search

Intuition: Recursively visit all vertices that are reachable along unvisited paths.

```
/** Visit all nodes reachable
on unvisited paths from u.
Precondition: u is unvisited.
*/
public static void dfs(int u)
{
    visited[u] = true;
    for all edges (u,v):
        if(!visited[v]):
            dfs(v);
}
```

Suppose there are n vertices that are reachable along unvisited paths and e edges:

Worst-case running time? $O(n + e)$

Worst-case space? $O(n)$

Depth-First Search in Java


```
public class Node {
        boolean visited;
        List<Node> neighbors;

/** Visit all nodes reachable on unvisited paths from
this node.
Precondition: this node is unvisited.
        public void dfs() {
                visited= true;
                for (Node n: neighbors) {
                        if (!n.visited) n.dfs();
                }
        }
}
```

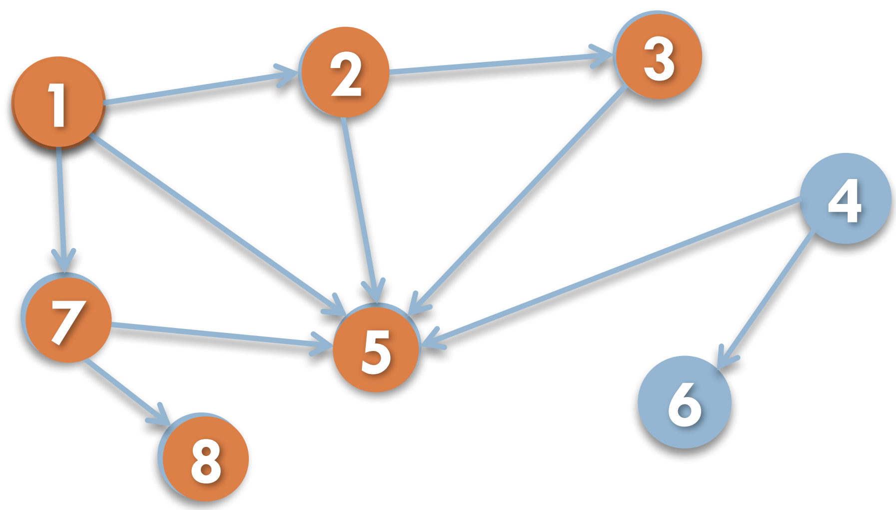
Each vertex of the graph is an object of type Node

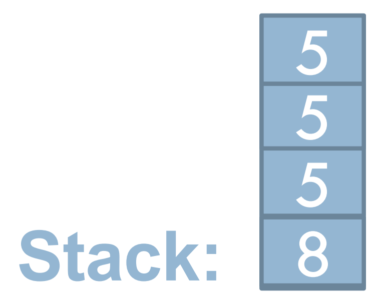
No need for a parameter. The object is the node.

Depth-First Search Iteratively

Intuition: Visit all vertices that are reachable along unvisited paths from the current node.

```
/** Visit all nodes reachable on
unvisited paths from u.
Precondition: u is unvisited. */
public static void dfs(int u) {
    Stack s= (u);// Not Java!
    while (s is not empty) {
        u= s.pop();
        if (u not visited) {
            visit u;
            for each edge (u, v):
                s.push(v);
        }
    }
}
```

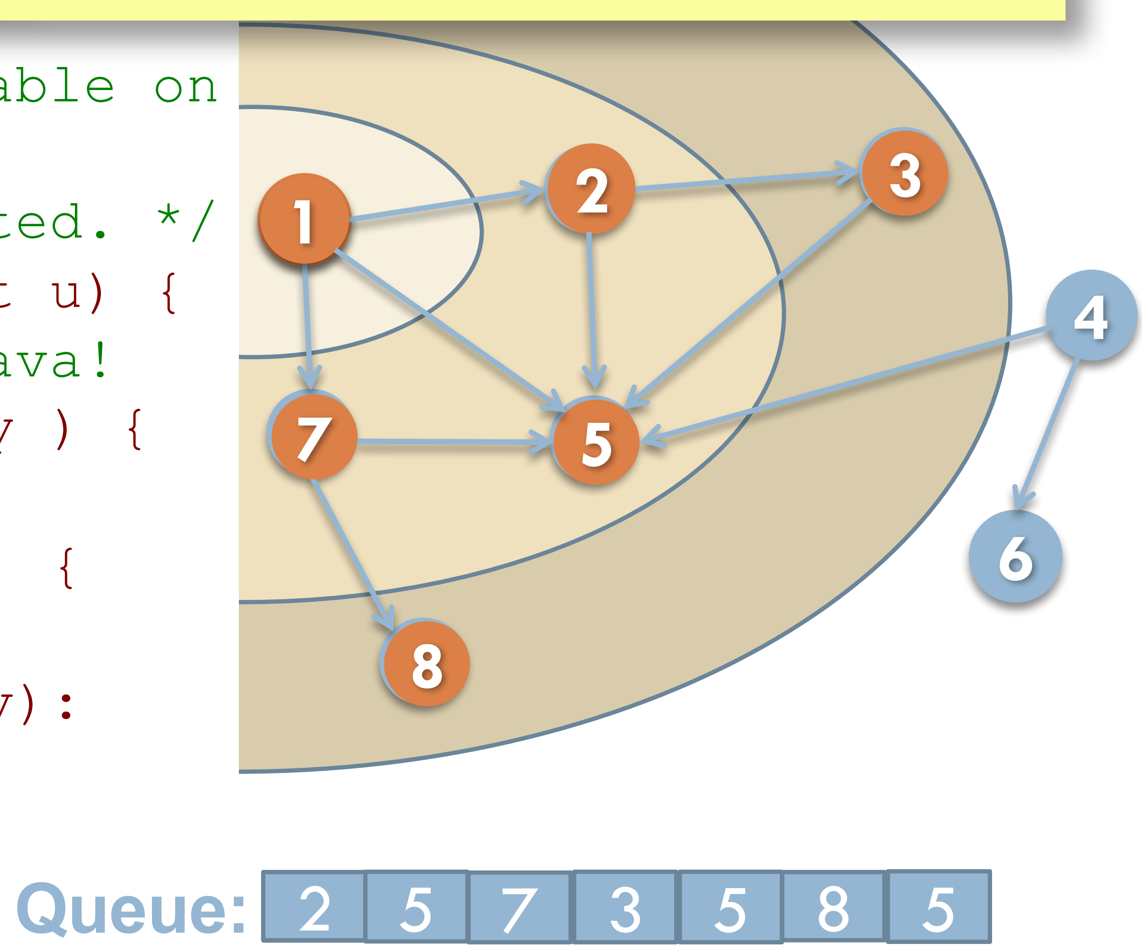




Breadth-First Search

Intuition: Iteratively process the graph in "layers" moving further away from the source node.

```
/** Visit all nodes reachable on
unvisited paths from u.
Precondition: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);// Not Java!
    while ( q is not empty ) {
        u= q.remove();
        if (u not visited) {
            visit u;
            for each (u, v):
                q.add(v);
            }
        }
}
```



Analyzing BFS

Intuition: Iteratively process the graph in "layers" moving further away from the source node.

```
/** Visit all nodes reachable on
unvisited paths from u.
Precondition: u is unvisited. */
public static void bfs(int u) {
    Queue q= (u);// Not Java!
    while (                ) {
        u= q.remove();
        if (u not visited) {
            visit u;
            for each (u, v):
                q.add(v);
            }
        }
}
```

Suppose there are n vertices that are reachable along unvisited paths and e edges:

Worst-case running time? $O(n + e)$

Worst-case space? $O(e)$

Comparing Search Algorithms

DFS

- Visits: 1, 2, 3, 5, 7, 8
- Time: $O(n + e)$
- Space: $O(n)$

BFS

- Visits: 1, 2, 5, 7, 3, 8
- Time: $O(n + e)$
- Space: $O(e)$