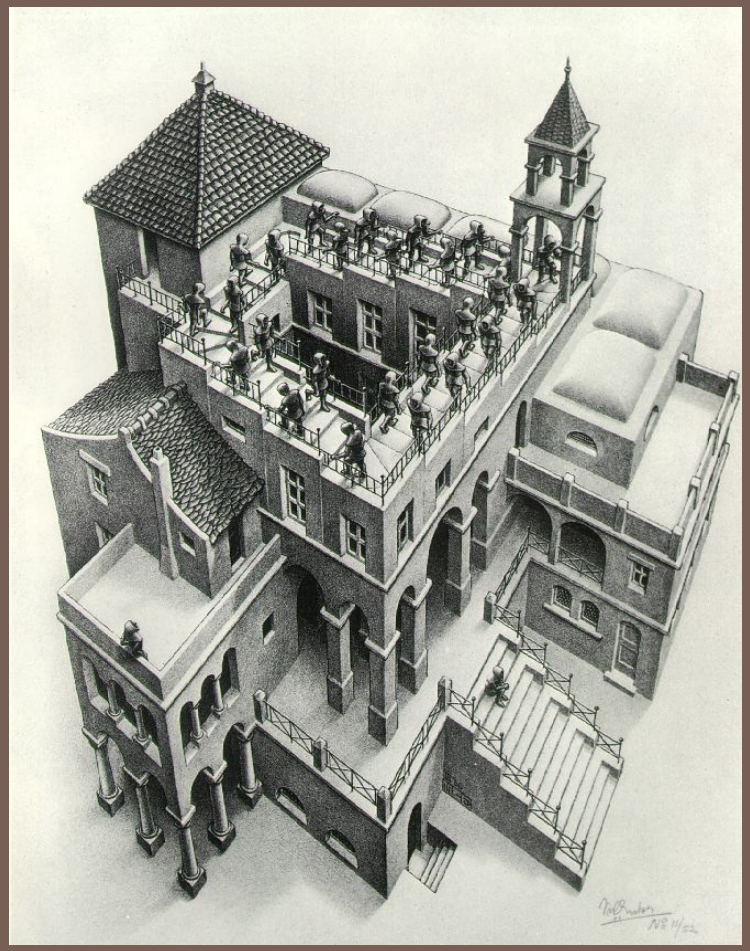


# HEAPS AND PRIORITY QUEUES

Lecture 16

CS2110 Fall 2017

# Abstract vs concrete data structures



- Abstract data structures are **interfaces**
  - they specify only **interface** (method names and specs)
  - not **implementation** (method bodies, fields, …)

- Concrete data structures are **classes**. Abstract data structures can have multiple possible implementations by different concrete data structures.

# Abstract vs concrete data structures



- **interface** List defines an “abstract data type”.
- It has methods: add, get, remove, …
- Various **classes** ("concrete data types") implement List:

| Class: | ArrayList | LinkedList |
|---|---|---|
| Backing storage: | array | chained nodes |
| add(i, val) | O(n) | O(n) |
| add(0, val) | O(n) | O(1) |
| add(n, val) | O(1) | O(1) |
| get(i) | O(1) | O(n) |
| get(0) | O(1) | O(1) |
| get(n) | O(1) | O(1) |

4

# Concrete Data Types

# Concrete data structures



- Array
- LinkedList (singley-linked, doubly-linked)
- Trees (binary, general)
- Heaps

# Heaps



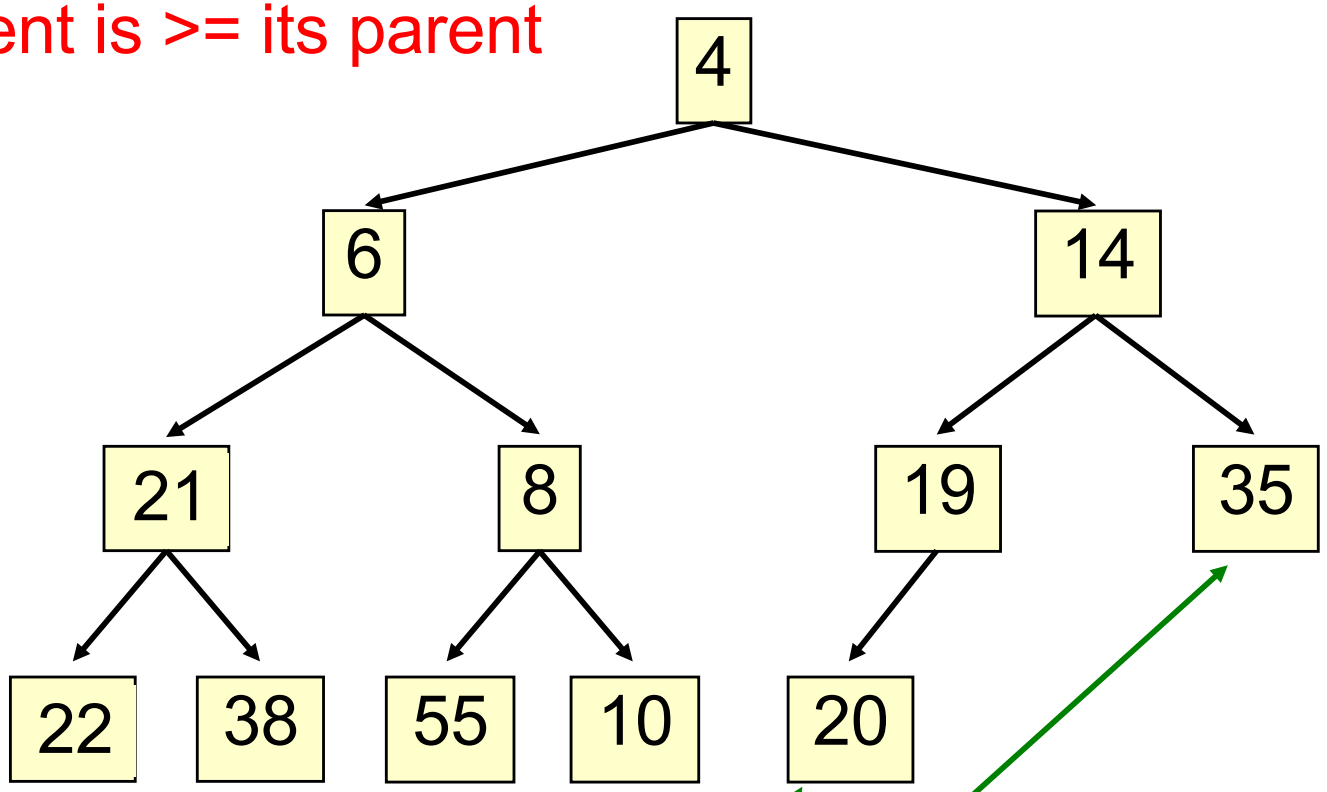
- A *heap* is a binary tree with certain properties (it's a concrete data structure)
  - Heap Order Invariant: every element in the tree is >= its parent
  - Complete Binary Tree: every level of the tree (except last) is completely filled, there are no holes

Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects – different usage of the word *heap*

# Order Property



Every element is >= its parent

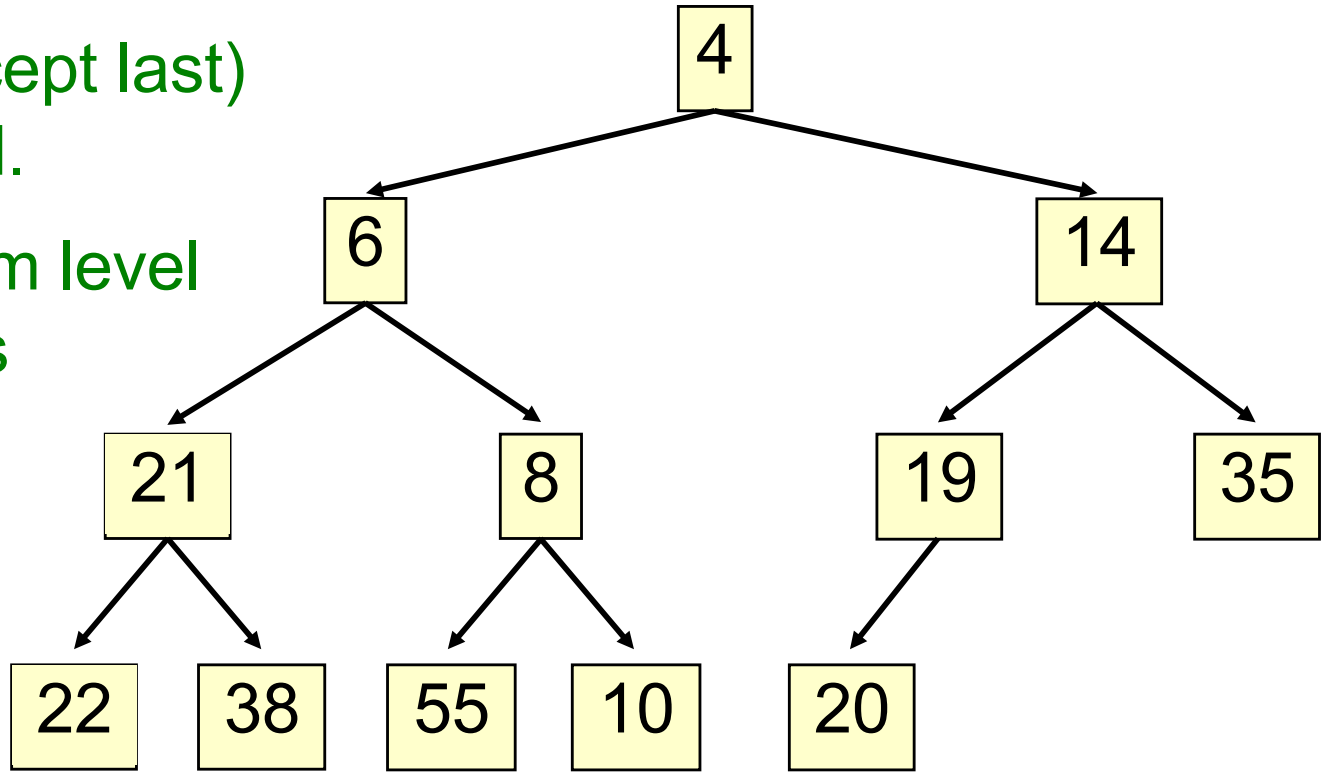



Note: 19, 20 < 35: Smaller elements can be deeper in the tree!

# Completeness Property

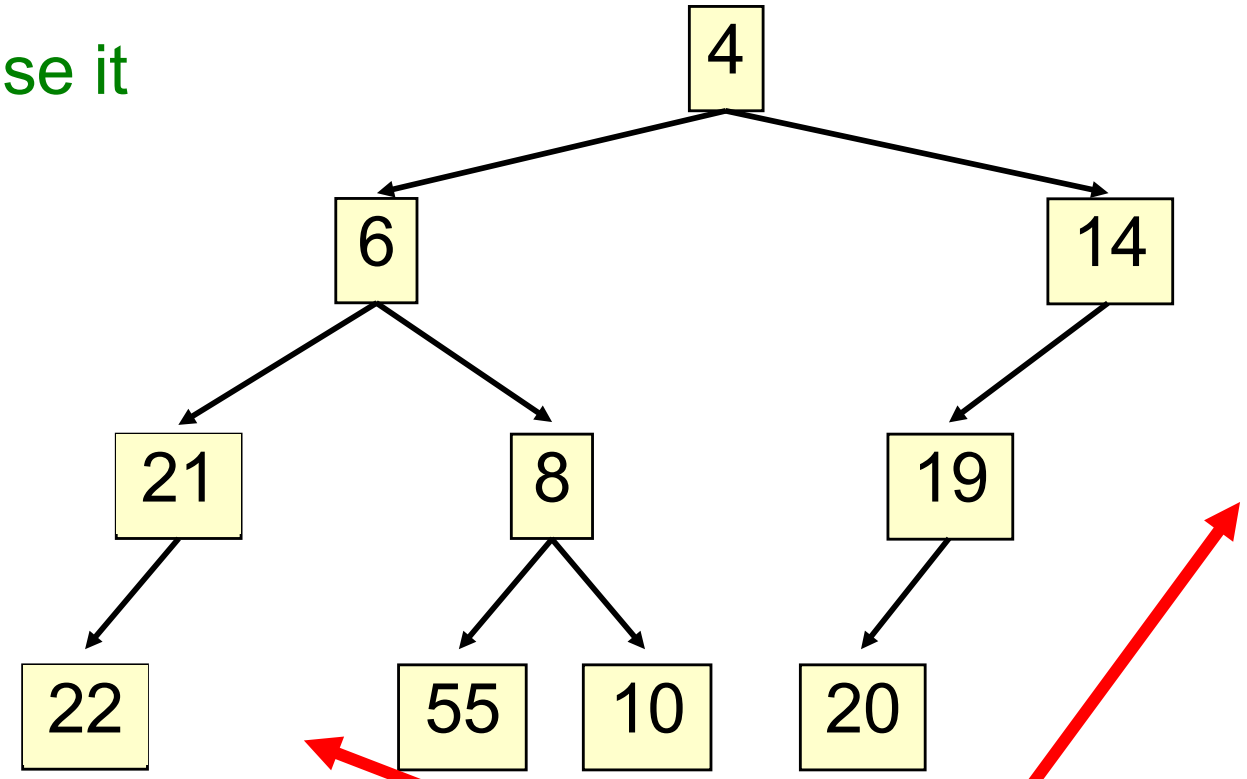
Every level (except last) completely filled.

Nodes on bottom level are as far left as possible.



# Completeness Property

Not a heap because it has two holes




Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible

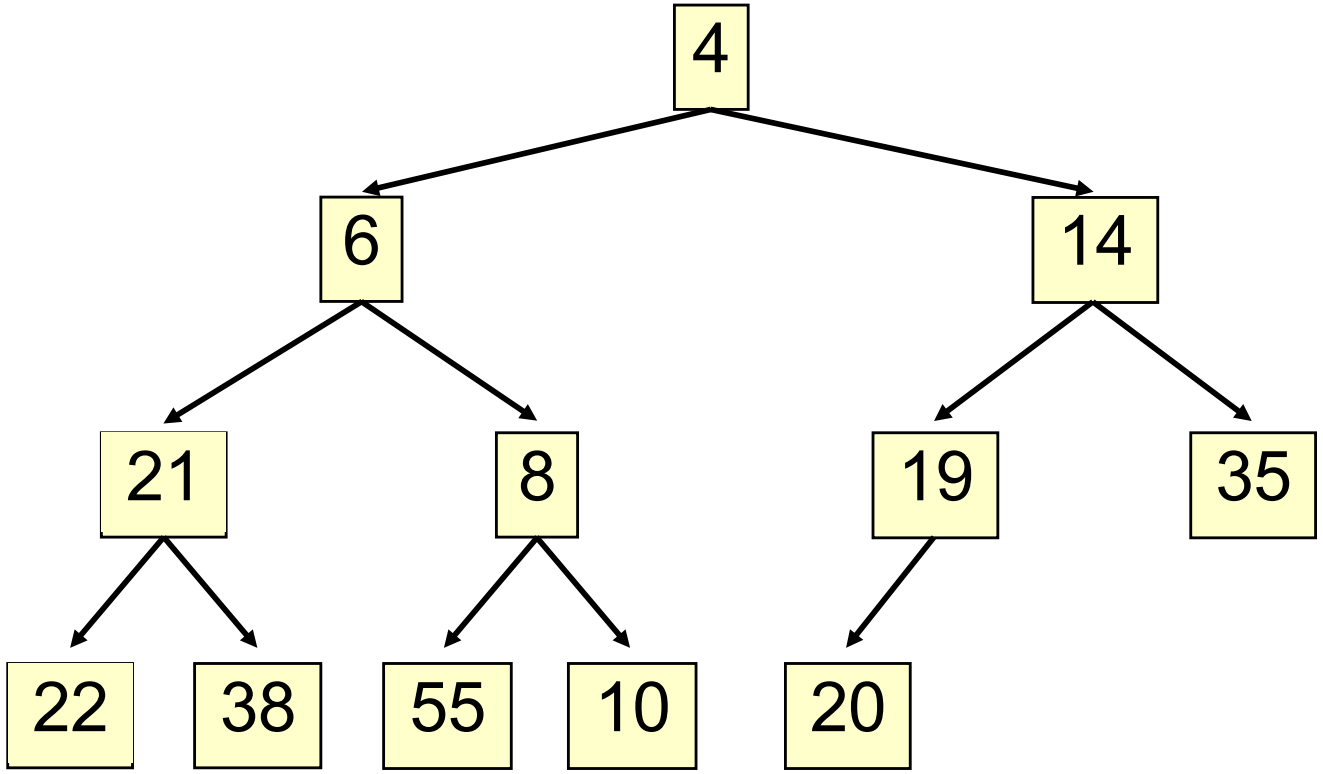
# Heaps



- A *heap* is a binary tree with certain properties (it's a concrete data structure)
  - Heap Order Invariant: every element in the tree is >= its parent
  - Complete Binary Tree: every level of the tree (except last) is completely filled, there are no holes
- A heap implements two key methods:
  - add(e): adds a new element to the heap
  - poll(): deletes the least element and returns it

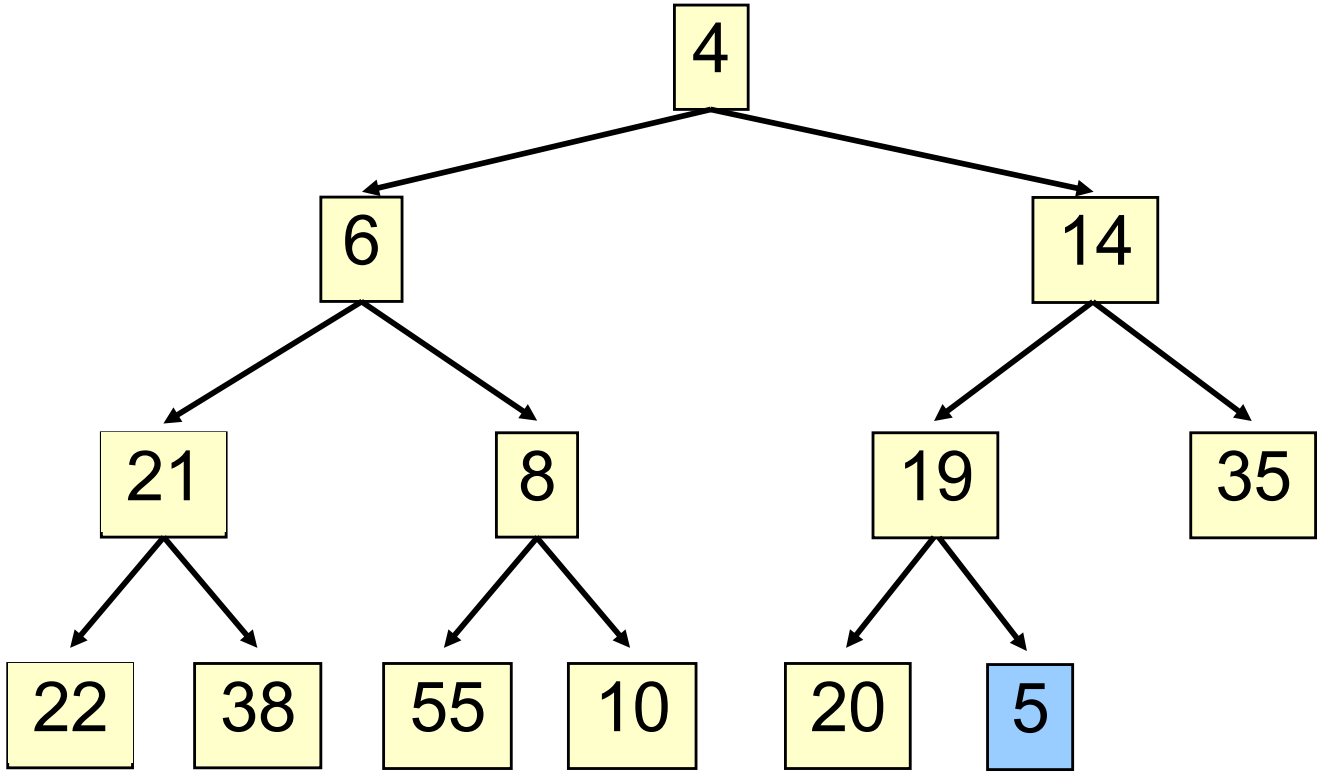
# add(e)





# add(e)

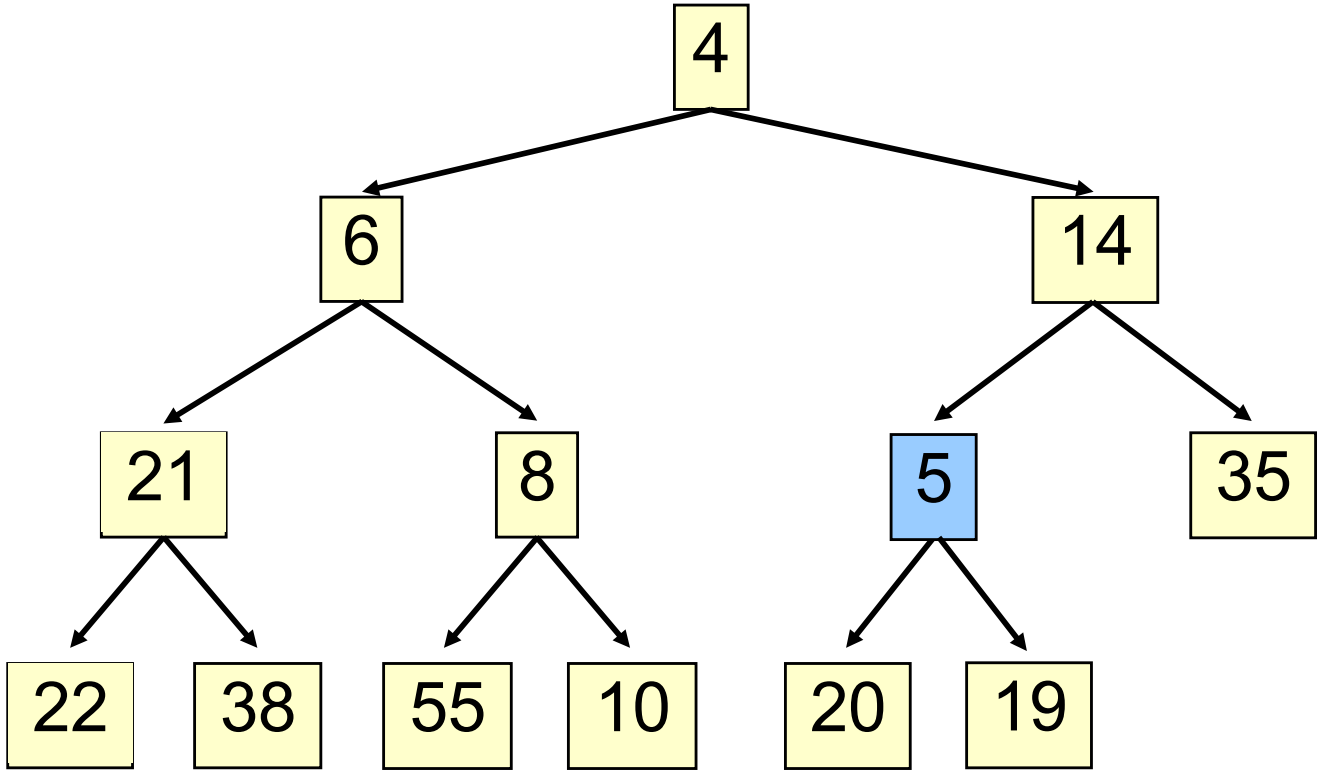





1. Put in the new element in a new node

# add()

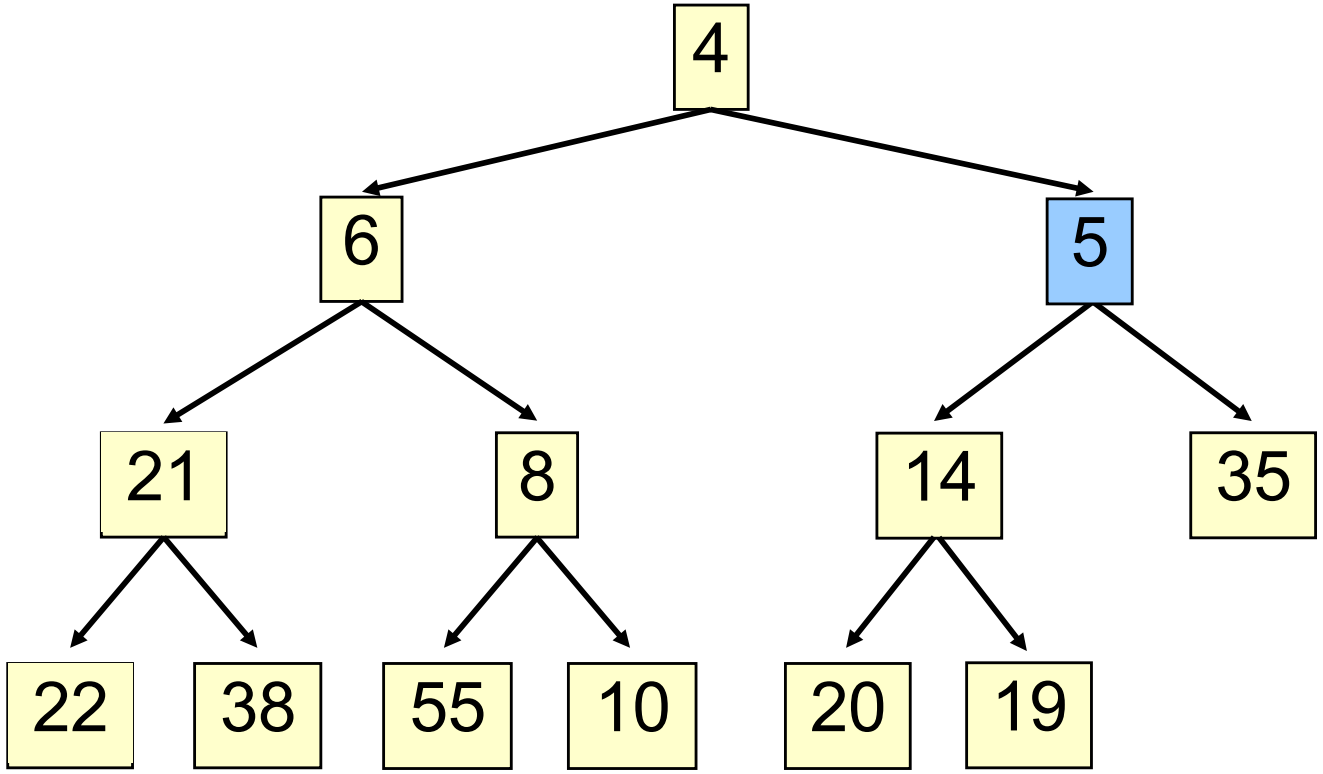





2. Bubble new element up if less than parent

# add()

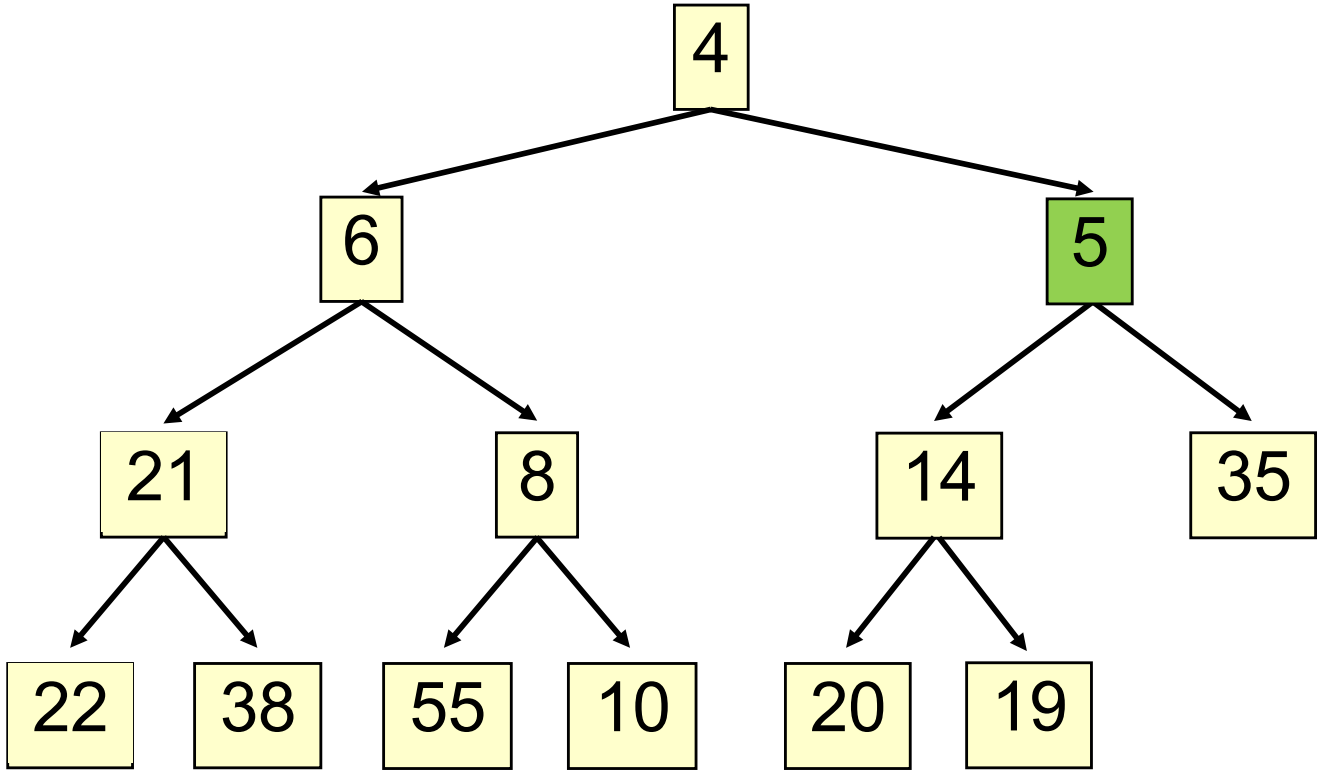





2. Bubble new element up if less than parent

# add()





# `add(e)`



- Add e at the leftmost empty leaf
- Bubble e up until it no longer violates heap order
- The heap invariant is maintained!

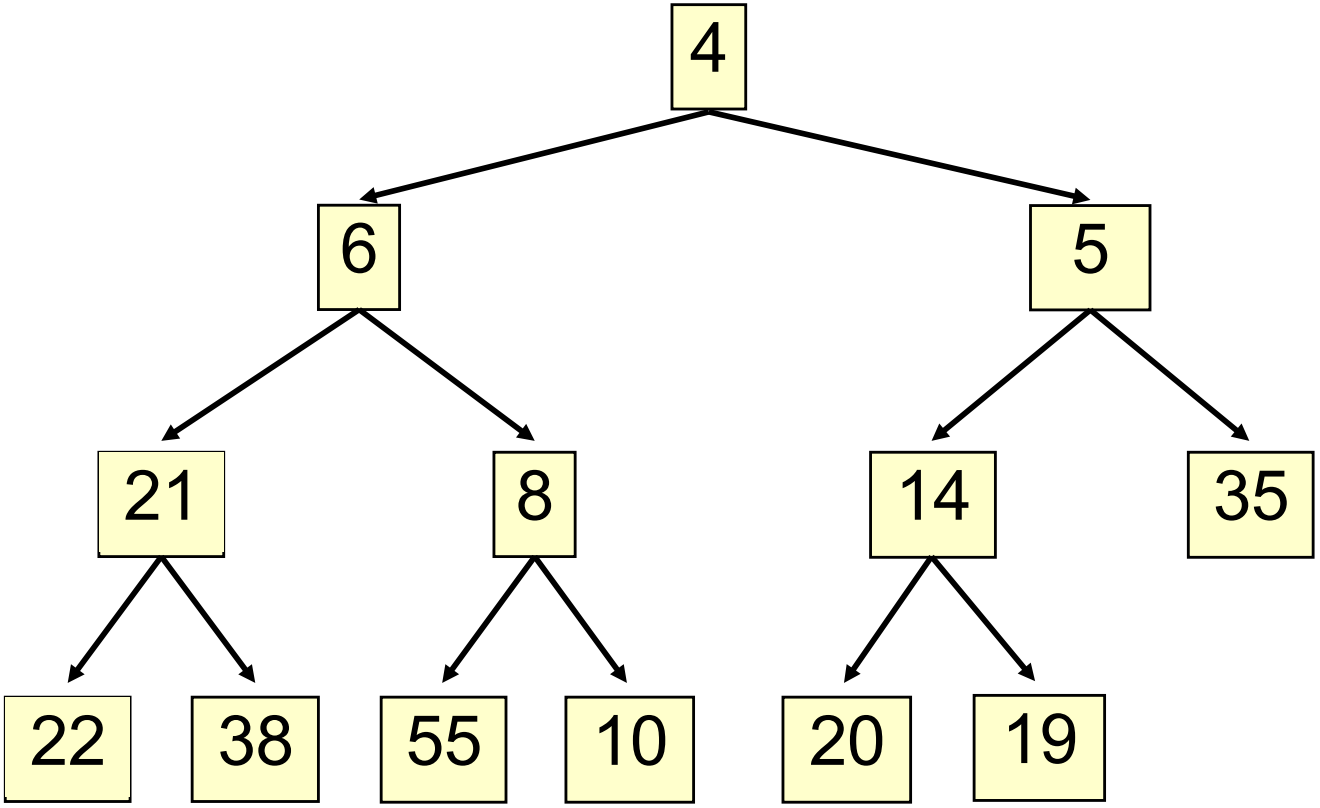
# `add(e)` to a tree of size n



- Time is O(log n), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size

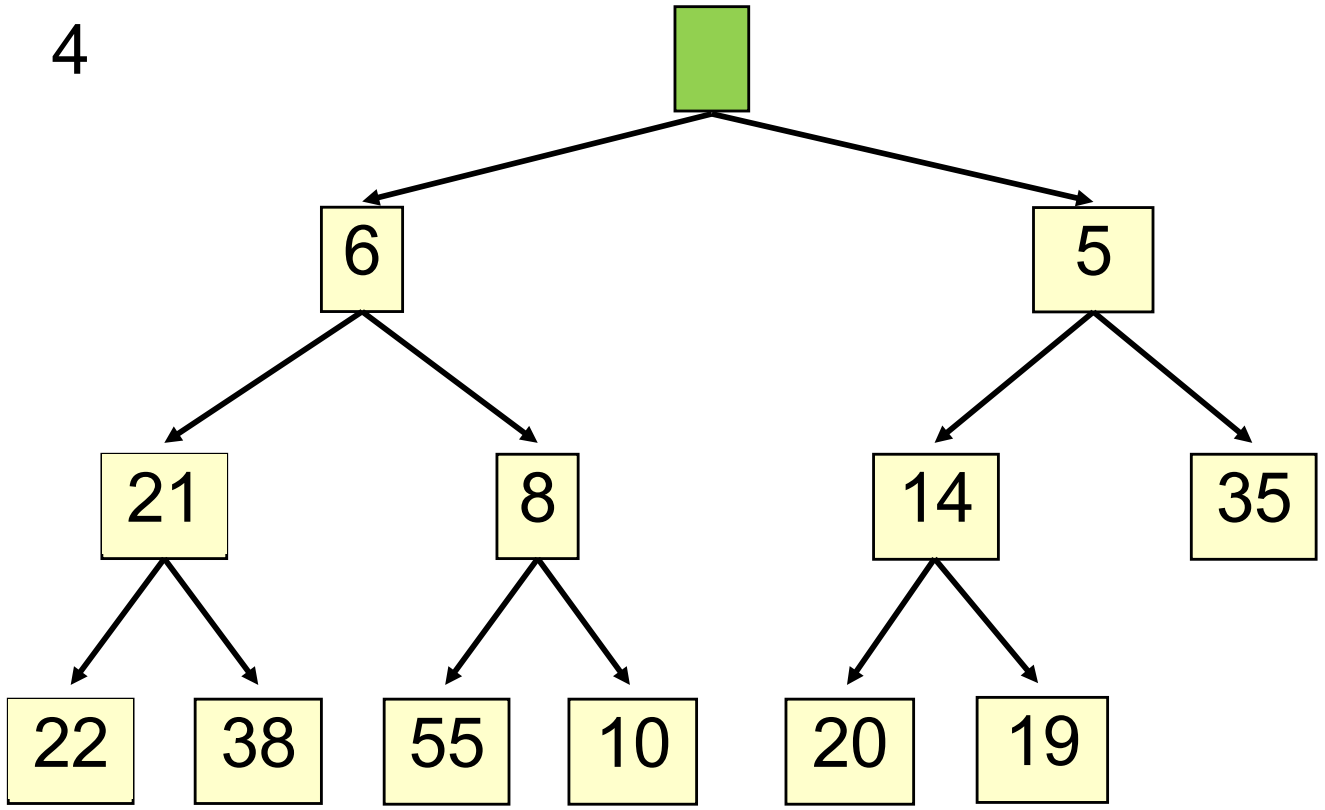
# poll()





# `poll()`

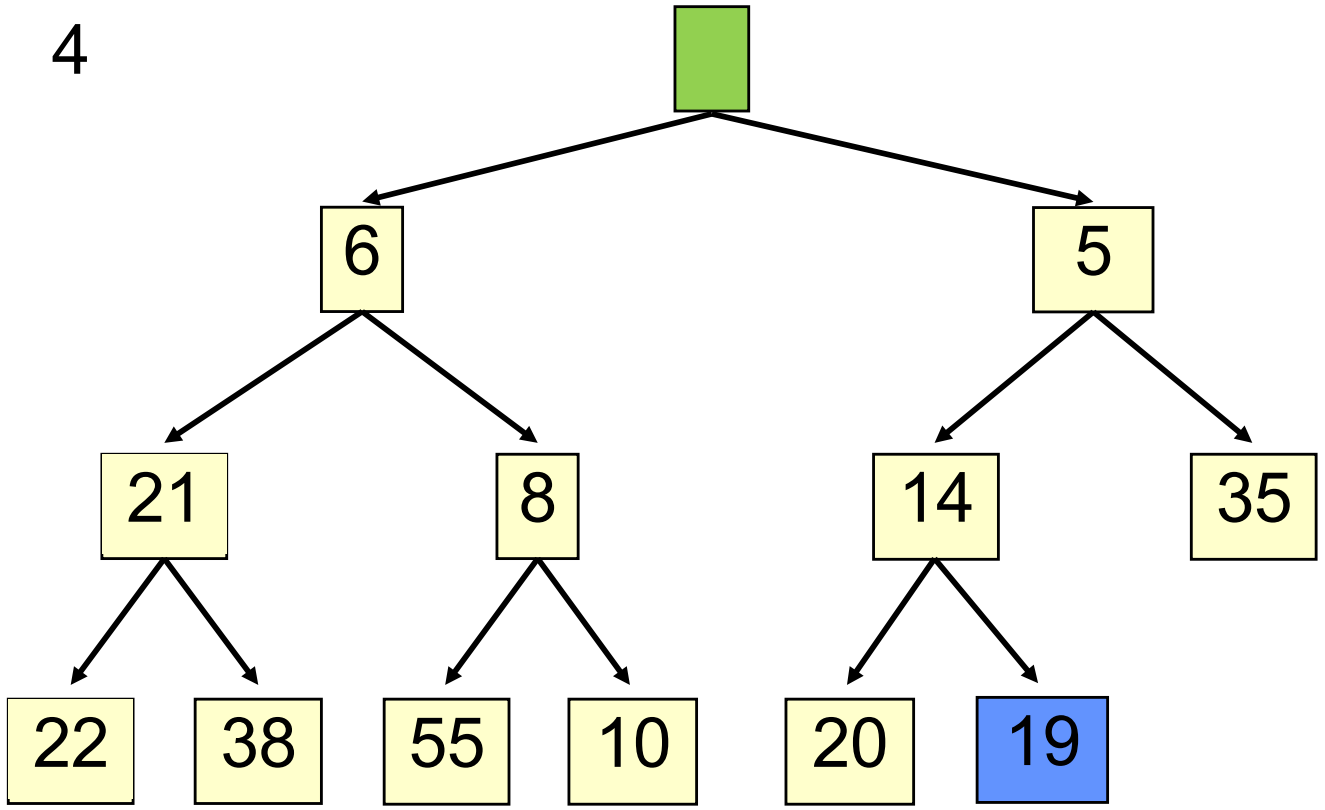





1. Save top element in a local variable

# poll()

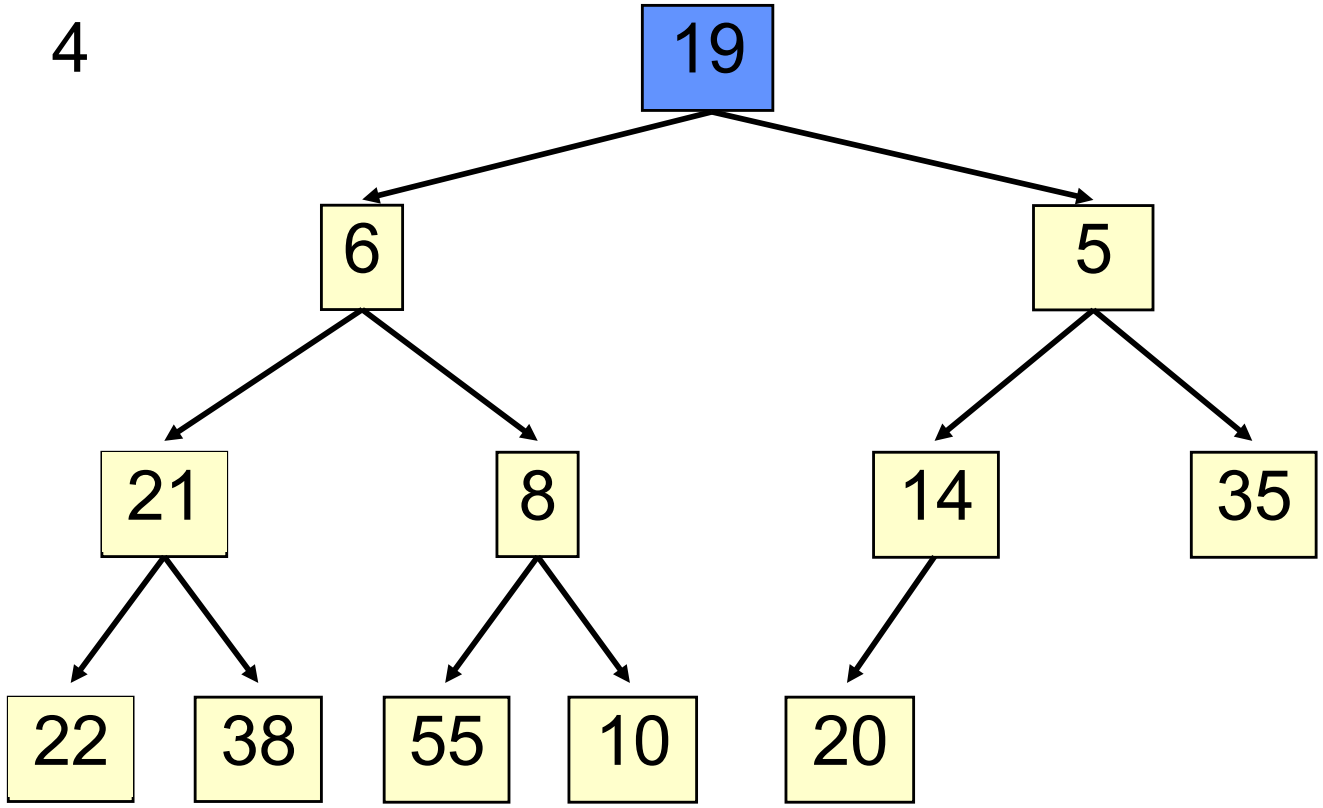





2. Assign last value to the root, delete last value from heap

# `poll()`

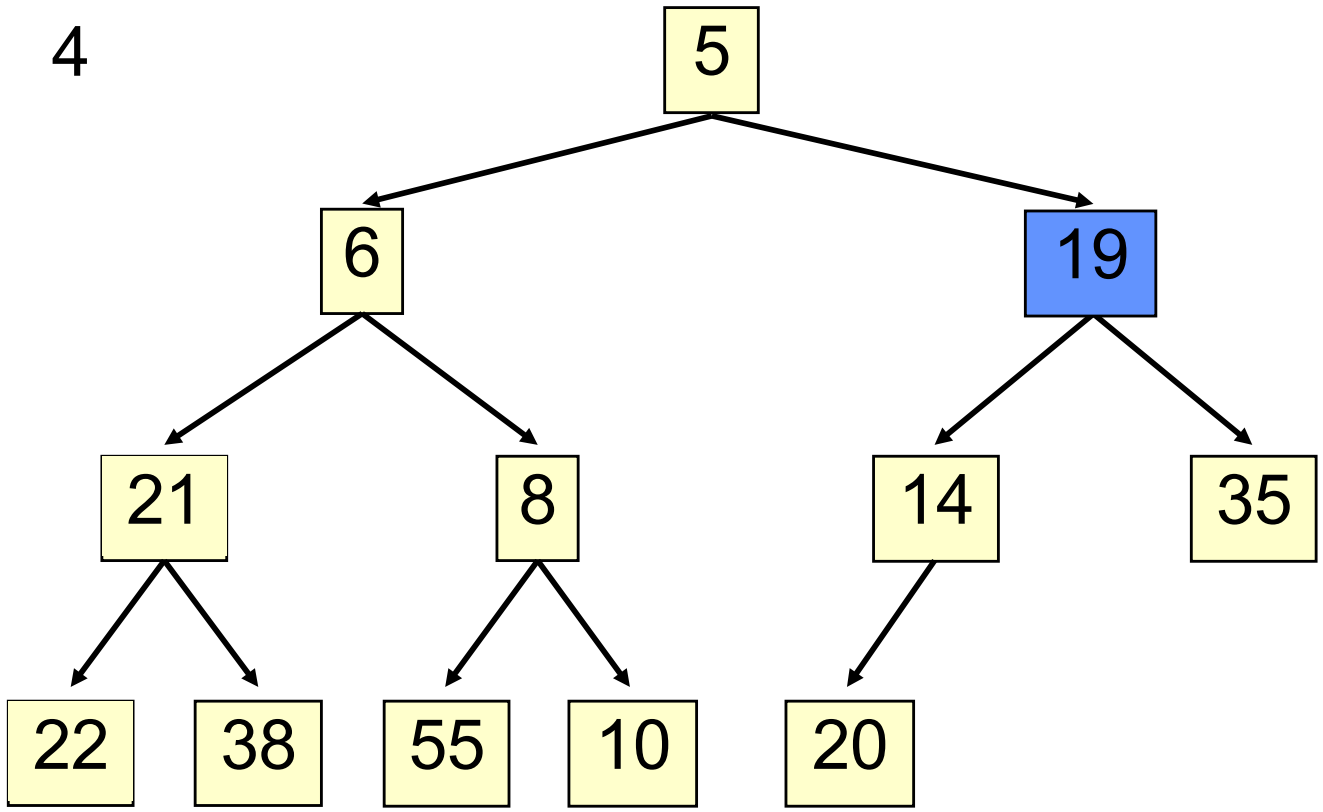





3. Bubble root value down

# poll()

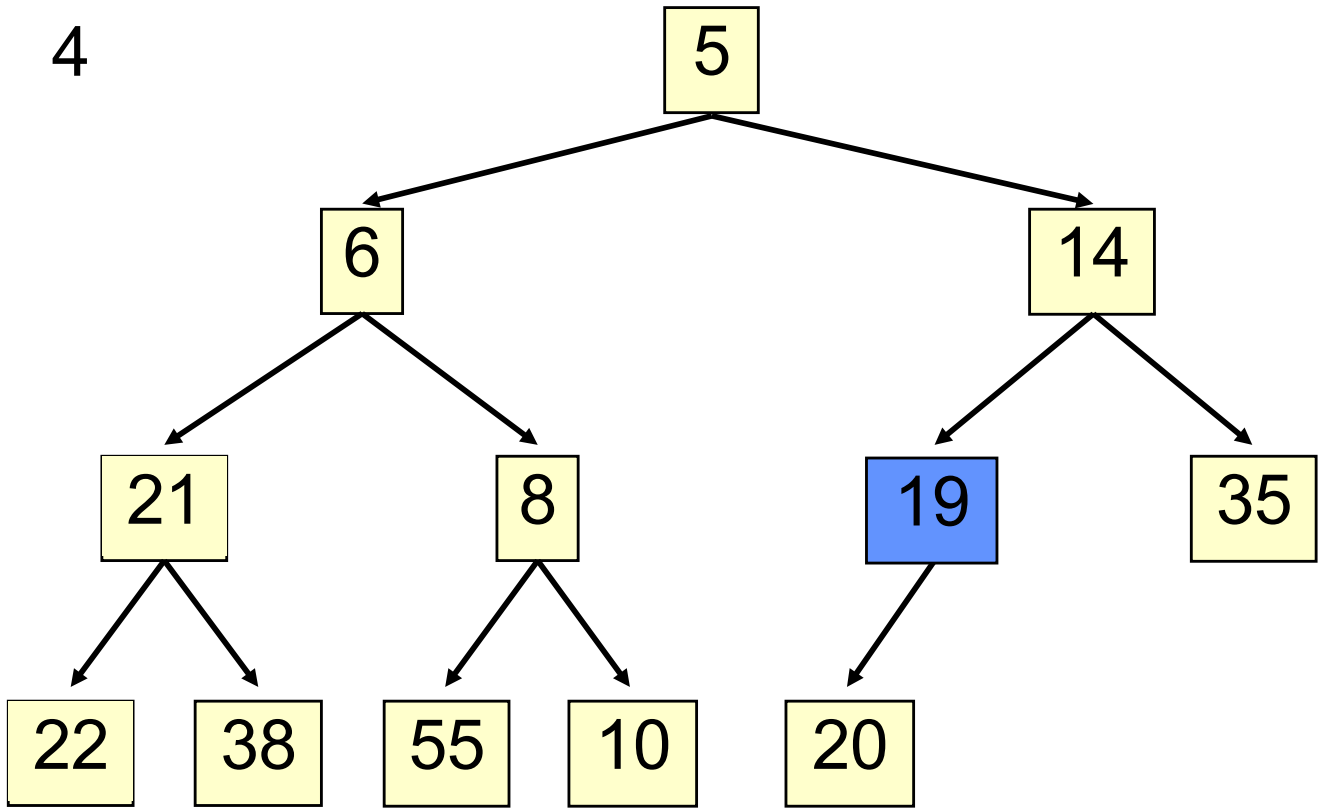





3. Bubble root value down

# poll()






3. Bubble root value down

# `poll()`



- Save the least element  (the root)
- Assign last element of the heap to the root.
- Remove last element of the heap.
- Bubble element down –always with smaller child, until heap invariant is true again.

The heap invariant is maintained!

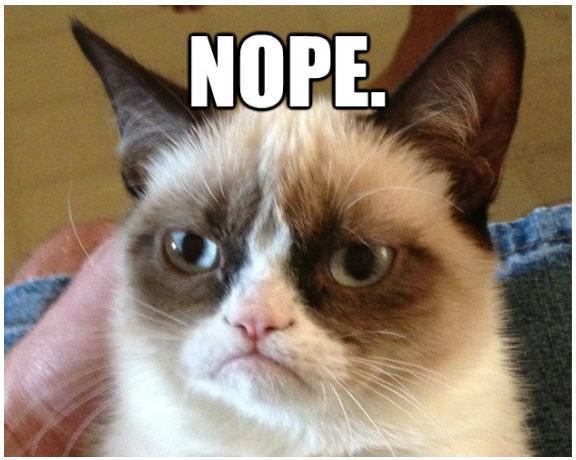
- Return the saved element

**Time is O(log n), since the tree is balanced**

# Implementing Heaps



```
public class HeapNode<E> {
   private E value;
   private HeapNode left;
   private HeapNode right;
   ...
}
```



# Implementing Heaps

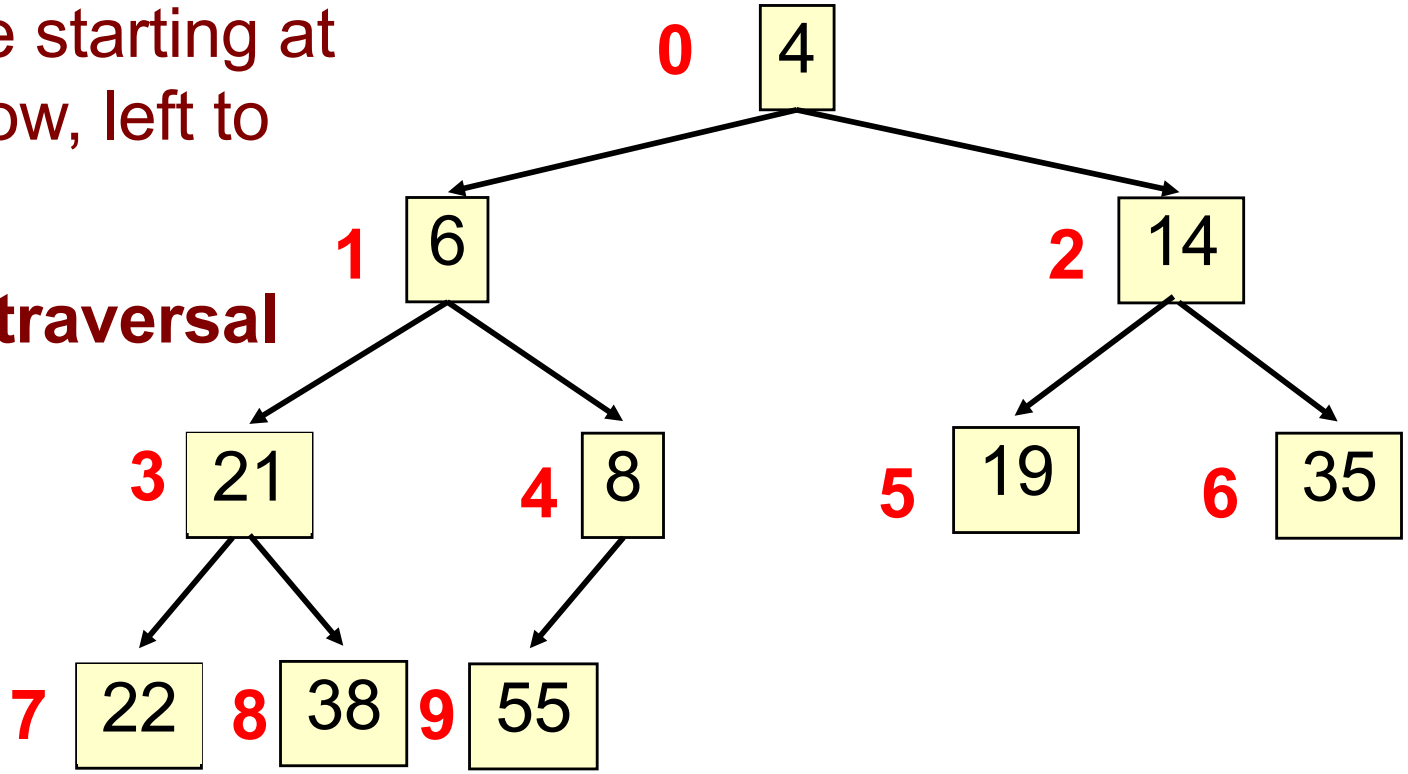


```
public class Heap<E> {
   private E[] heap;
   ...
}
```

# Numbering the nodes in a heap

Number node starting at root row by row, left to right

**Level-order traversal**

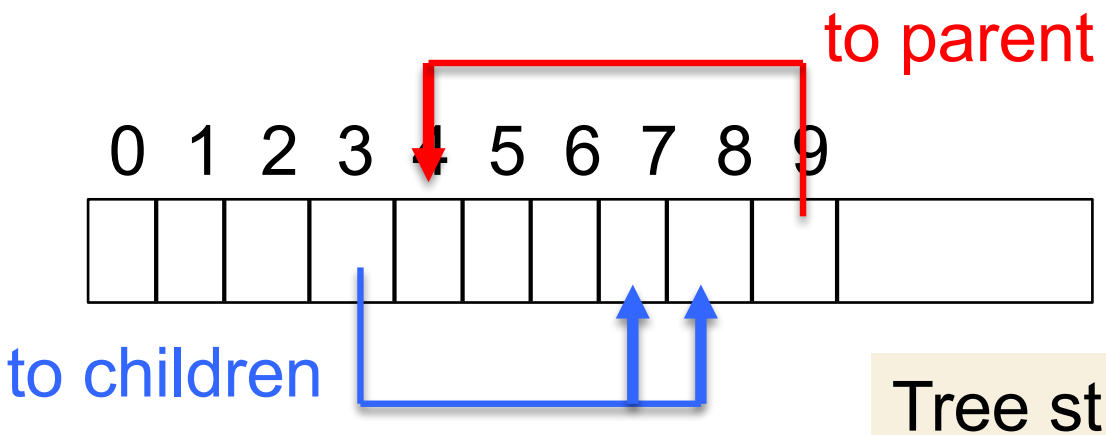



Children of node k are nodes 2k+1 and 2k+2
Parent of node k is node (k-1)/2

# Store a heap in an array (or ArrayList) b!



- Heap nodes in b in order, going across each level from left to right, top to bottom
- Children of b[k] are b[2k + 1] and b[2k + 2]
- Parent of b[k] is b[(k – 1)/2]




Tree structure is implicit.
No need for explicit links!

# add()  --assuming there is space



```
/** An instance of a heap */
class Heap<E> {
  E[] b= new E[50];  // heap is b[0..n-1]
  int n= 0;          // heap invariant is true

  /** Add e to the heap */
  public void add(E e) {
    b[n]= e;
    n= n + 1;
    bubbleUp(n - 1); // given on next slide
  }
}
```

# add(). Remember, heap is in b[0..n-1]



```
class Heap<E> {
  /** Bubble element #k up to its position.
    * Pre: heap inv holds except maybe for k */
  private void bubbleUp(int k) {
    int p= (k-1)/2;
    // inv: p is parent of k and every elmnt
    // except perhaps k is >= its parent
    while (k > 0  &&  b[k].compareTo(b[p]) < 0) {
        swap(b[k], b[p]);
        k= p;
        p= (k-1)/2;
    }
}
```

# poll(). Remember, heap is in b[0..n-1]



```
/** Remove and return the smallest element
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v=  b[0];   // smallest value at root.
    n= n - 1;     // move last
    b[0]= b[n];   // element to root
    bubbleDown(0);
    return v;
}
```

# c's smaller child



```
/** Tree has n node.
 *  Return index of smaller child of node k
    (2k+2 if k >= n) */
public int smallerChild(int k, int n) {
   int c= 2*k + 2;      // k's right child
   if (c >= n || b[c-1].compareTo(b[c]) < 0)
      c= c-1;
   return c;
}
```

```
/** Bubble root down to its heap position.
    Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k= 0;
    int c= smallerChild(k, n);
   // inv: b[0..n-1] is a heap except maybe b[k] AND
   //      b[c] is b[k]'s smallest child
   while ( c < n &&  b[k].compareTo(b[c]) > 0) {

       swap(b[k], b[c]);
       k= c;
       c= smallerChild(k, n);

   }
}
```

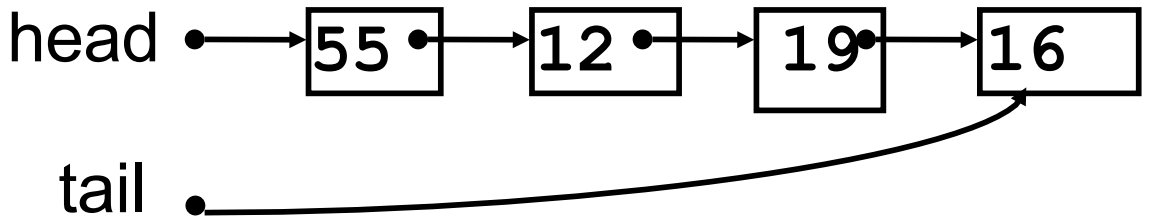
# Abstract Data Types

# Some Abstract Data Types



- List
- Stack (LIFO) implemented using a List
  - allows only **`add(0,val)`**, **`remove(0)`** (push, pop)
- Queue (FIFO) implemented using a List
  - allows only **`add(n,val)`**, **`remove(0)`** (enqueue, dequeue)

Both efficiently implementable using a singly linked list with head and tail




- PriorityQueue

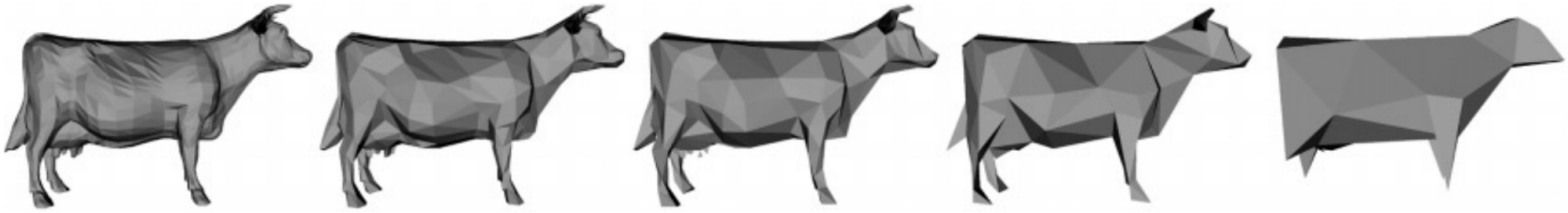
# Priority Queue



- Data structure in which data items are `Comparable`
- Smaller elements (determined by `compareTo()`) have higher priority
- `remove()` return the element with the highest priority = least element in the `compareTo()` ordering
- break ties arbitrarily

# Many uses of priority queues (& heaps)





Surface simplification [Garland and Heckbert 1997]

- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest M values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling
- College: prioritizing assignments for multiple classes.

# java.util.PriorityQueue<E>



```
interface PriorityQueue<E> {
 boolean add(E e) {...} //insert e.
 void clear() {...} //remove all elems.
 E peek() {...} //return min elem.
 E poll() {...} //remove/return min elem.
 boolean contains(E e)
 boolean remove(E e)
 int size() {...}
 Iterator<E> iterator()
}
```

| | TIME |
|---|---|
| add | log |
| clear | |
| peek | constant |
| poll | log |
| contains | linear |
| remove | linear |
| size | constant |
| iterator | |

IF implemented with a heap!

# Priority queues as lists



- Maintain as a list
  - **`add()`** put new element at front – O(1)
  - **poll()** must search the list – O(n)
  - **peek()** must search the list – O(n)

- Maintain as an ordered list
  - **`add()`** must search the list – O(n)
  - **poll()** min element at front – O(1)
  - **peek()** O(1)

Can we do better?

# Priority queues as heaps



- A *heap* is can be used to implement priority queues
- Gives better complexity than either ordered or unordered list implementation:
  - **`add()`**: O(log n) (n is the size of the heap)
  - **`peek()`**: O(1)
  - **`poll()`**: O(log n)

# What if the priority is independent from the value?



Separate priority from value and do this:

```
add(e, p);  //add element e with priority p (a double)
```

THIS IS EASY!

Be able to change priority

```
change(e, p);  //change priority of e to p
```

THIS IS HARD!

Big question: How do we find e in the heap?
Searching heap takes time proportional to its size! No good!
Once found, change priority and bubble up or down. OKAY

Assignment A6: implement this heap! Use a second data structure to make change-priority expected log n time