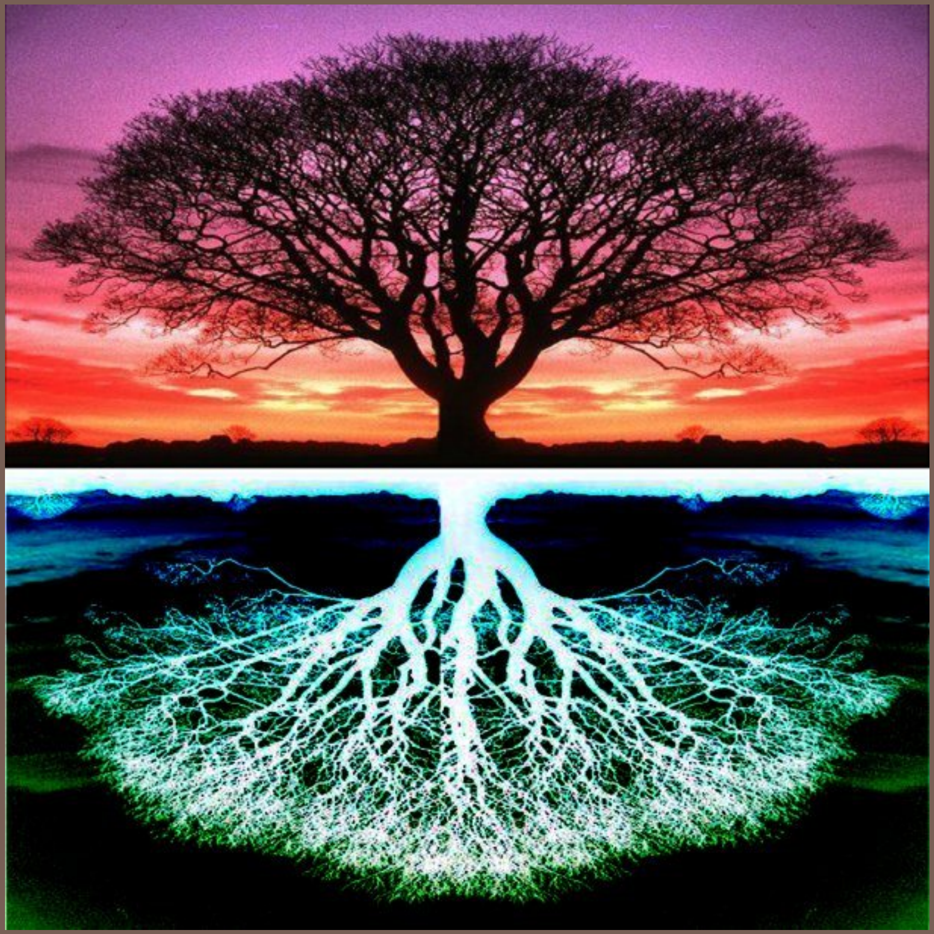


# TREES

Lecture 12

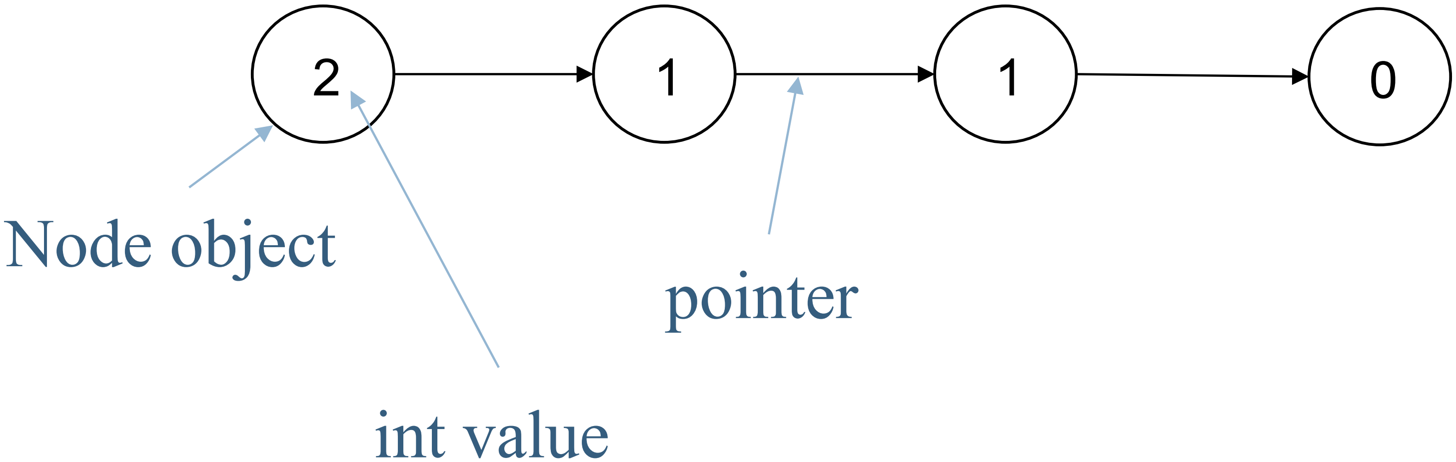
CS2110 – Fall 2017

# Important Announcements

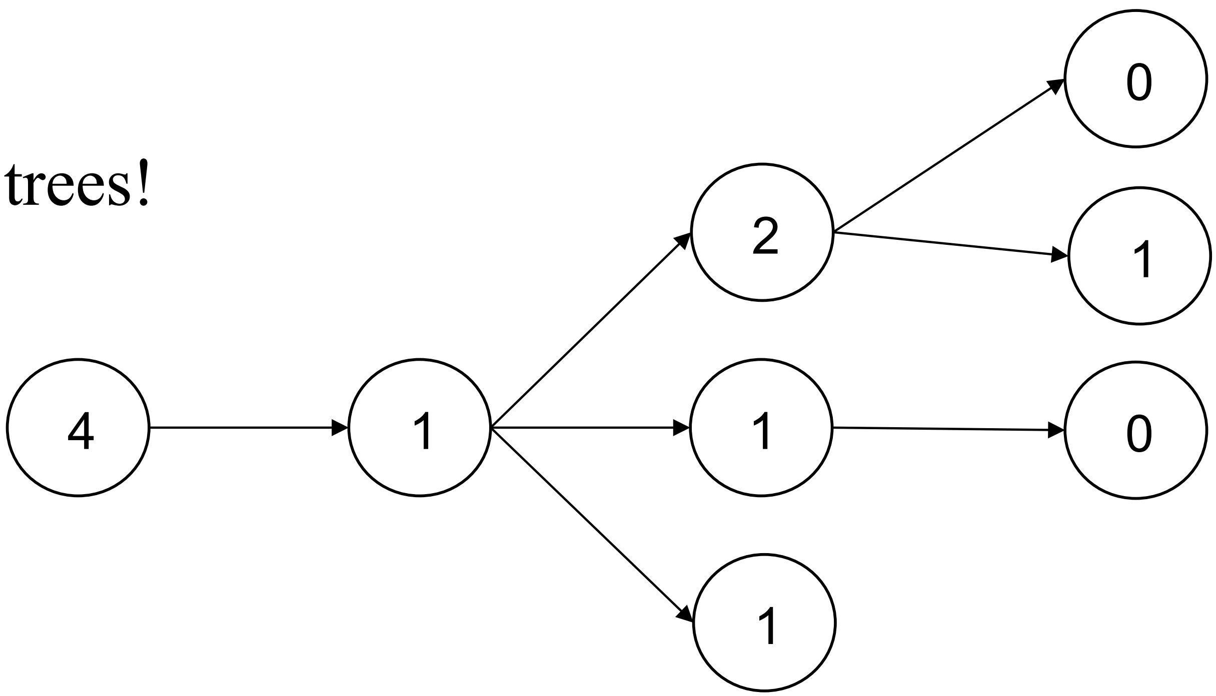


- A4 is out now and due two weeks from today. Have fun, and start early!

A picture of a singly linked list:




Today: trees!

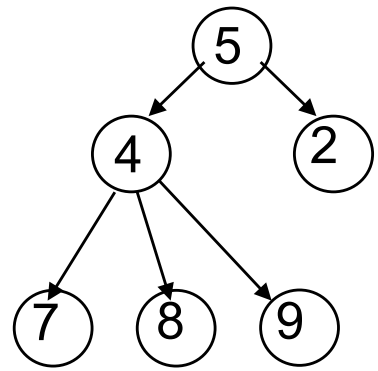


# Tree Overview

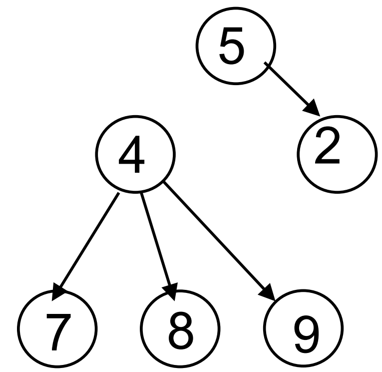


*Tree*: data structure with nodes, similar to linked list

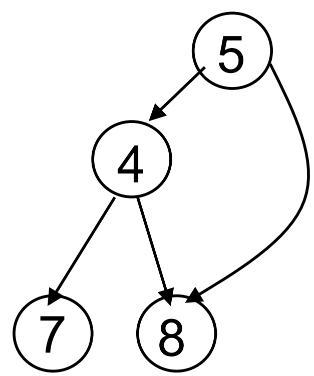
- Each node may have zero or more *successors* (children)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from *root*



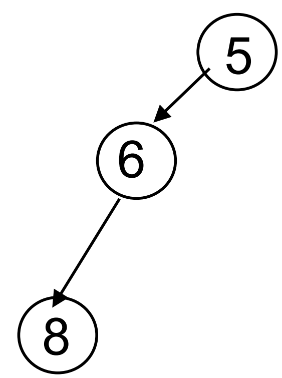

A tree




Not a tree




Also not a tree



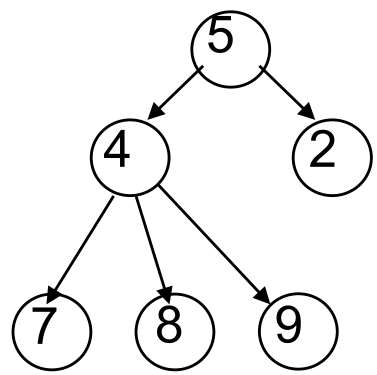

List-like tree

# Binary Trees

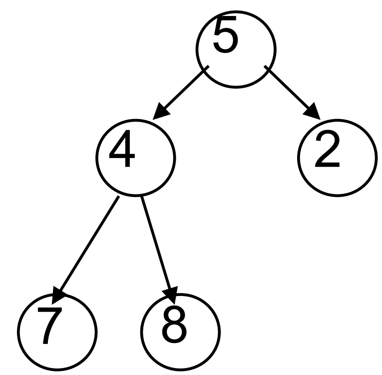


A *binary tree* is a particularly important kind of tree where every node as at most two children.

In a binary tree, the two children are called the *left* and *right* children.




Not a binary tree
(a *general* tree)



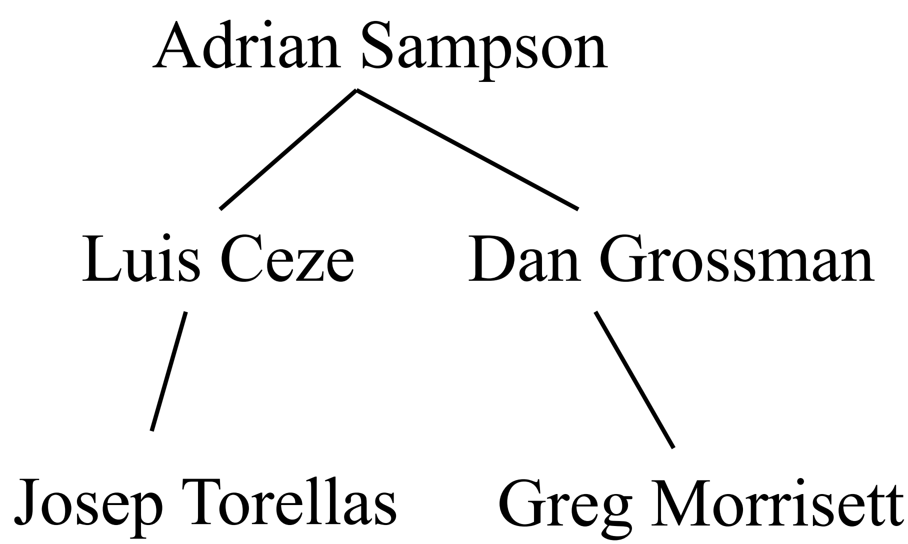

Binary tree

# Binary trees were in A1!



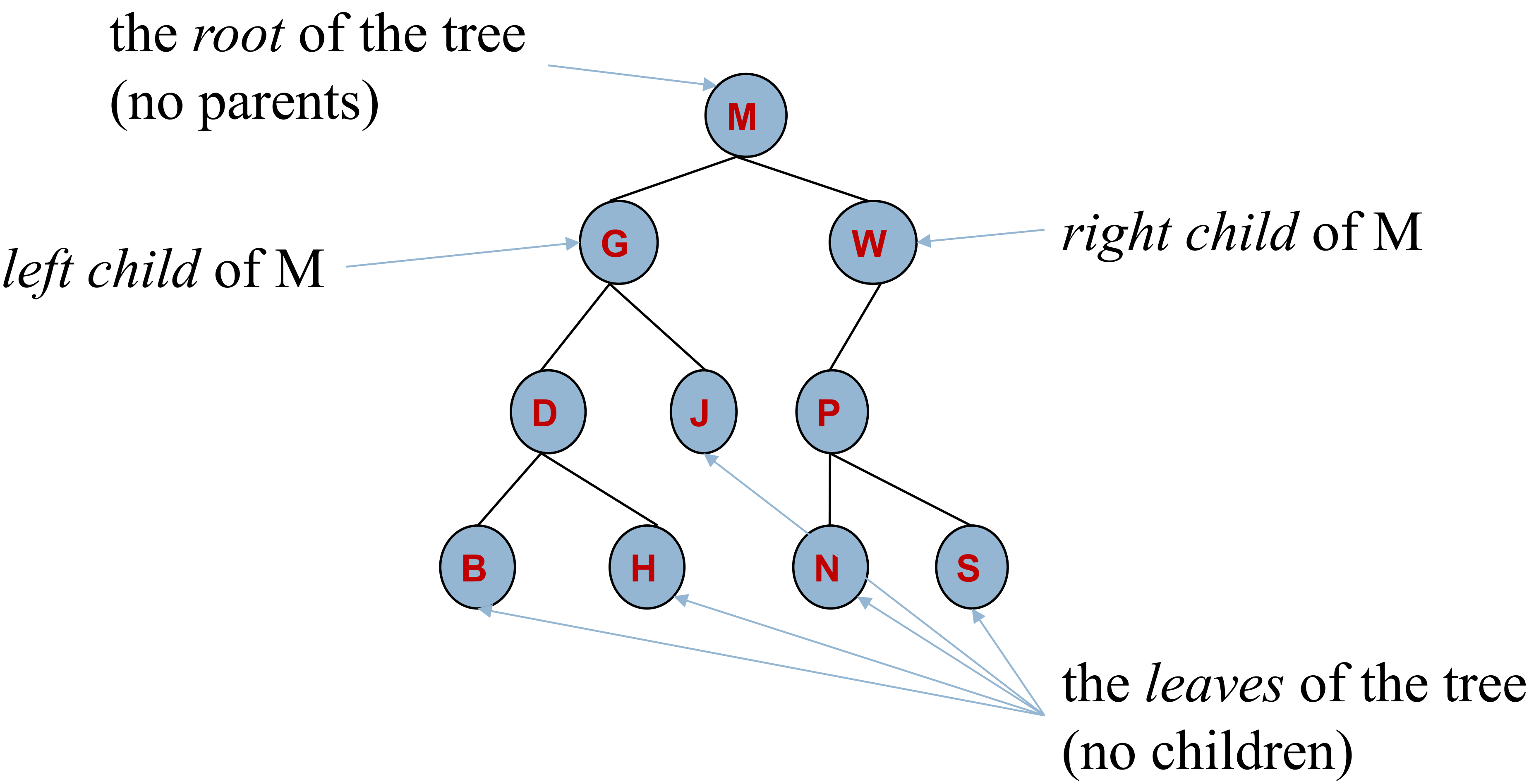
You have seen a binary tree in A1.

A PhD object has one or two advisors. (Confusingly, my advisors are my "children.")



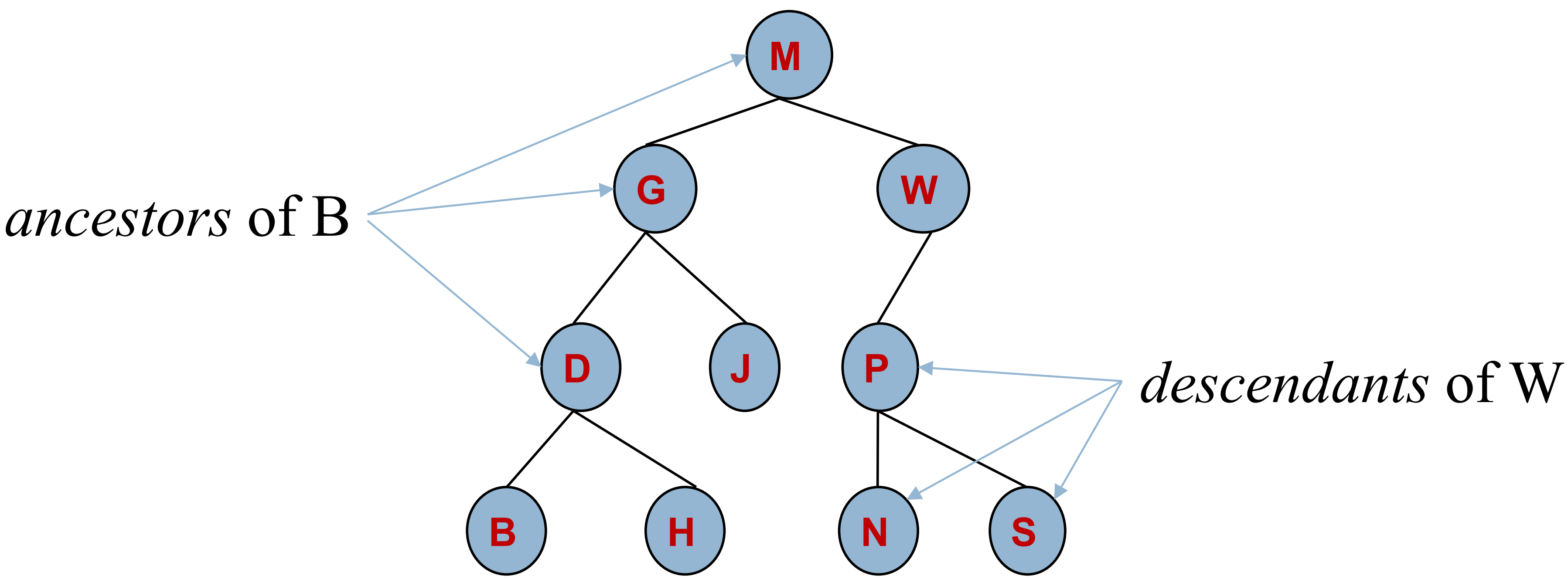
# Tree Terminology





# Tree Terminology

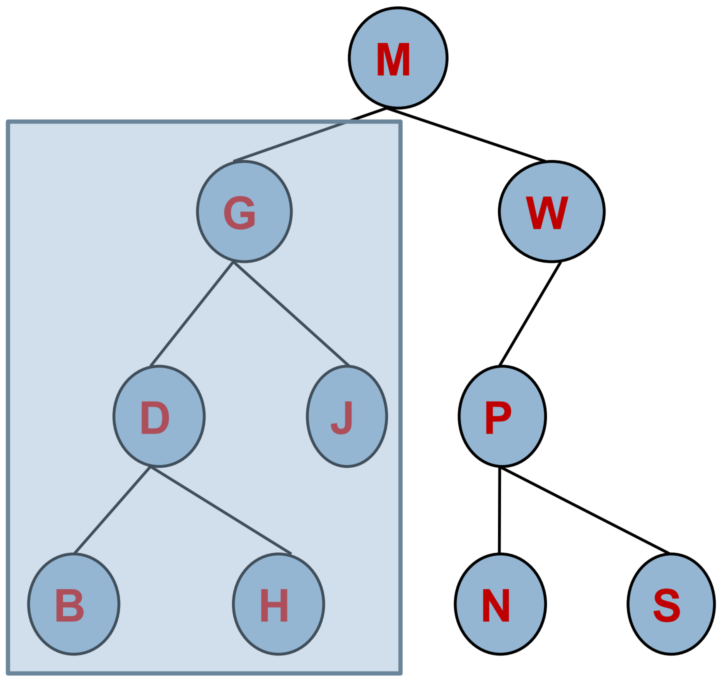




# Tree Terminology



*left subtree* of M

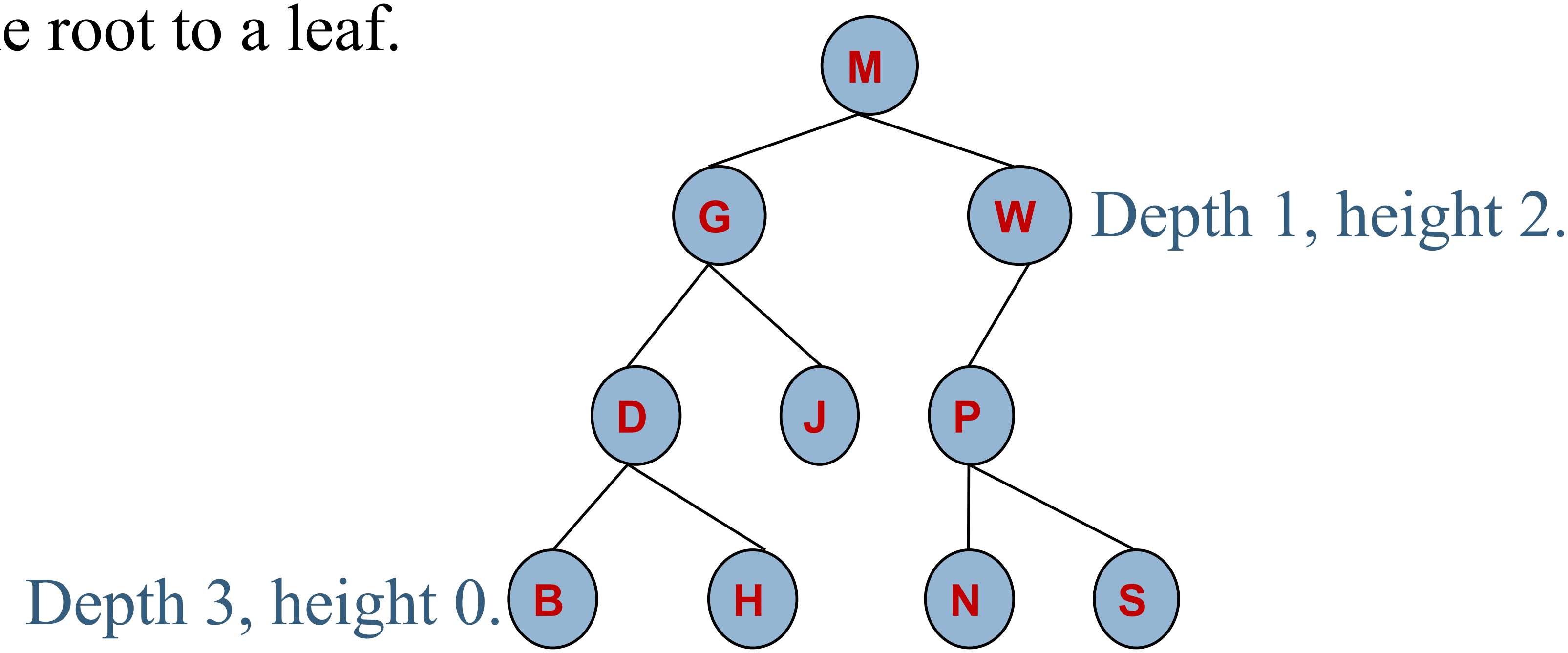


# Tree Terminology



A node's *depth* is the length of the path to the root.

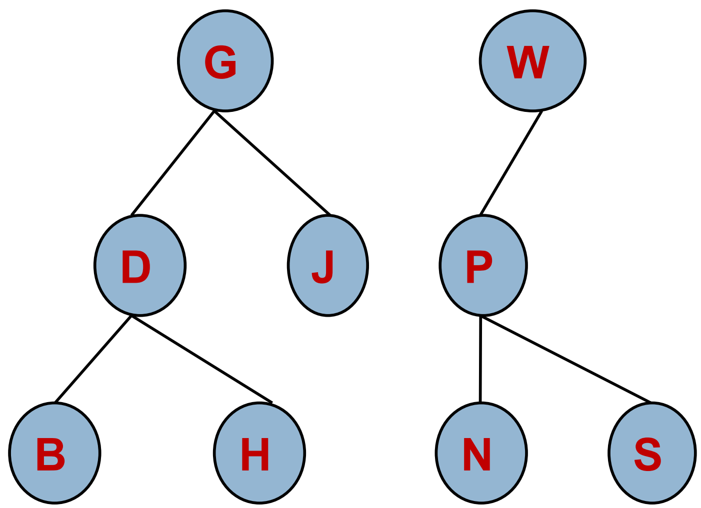
A tree's (or subtree's) *height* is he length of the longest path from the root to a leaf.



# Tree Terminology



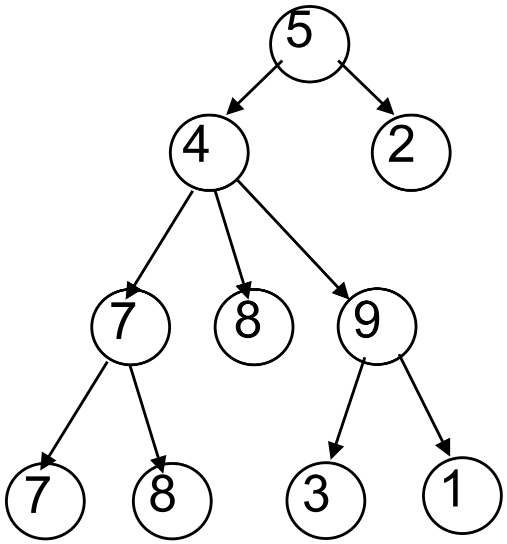
Multiple trees: a *forest*.



# Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Parent contains a list of its children




General tree

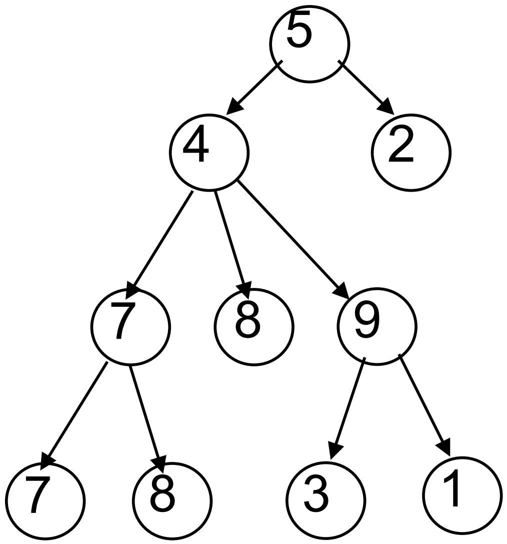
# Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Java.util.List is an interface!

It defines the methods that all implementation must implement.

Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?




General tree

# Class for binary tree node



```
class TreeNode<T> {
  private T value;
  private TreeNode<T> left, right;

  /** Constructor: one-node tree with datum x */
  public TreeNode (T d) { datum= d; left= null; right= null;}

  /** Constr: Tree with root value x, left tree l, right tree r */
  public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
     datum= d; left= l; right= r;
  }
}
```

Either might be null if the subtree is empty.

more methods: getValue, setValue, getLeft, setLeft, etc.

# Binary versus general tree



In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be **null**, meaning the subtree is empty (remember, a tree is a set of nodes)

In a general tree, a node can have any number of child nodes (and they need not be ordered)

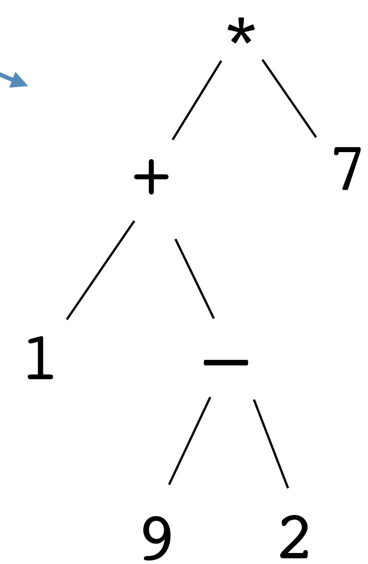
- Very useful in some situations ...
- ... one of which may be in an assignment!

# An Application: Syntax Trees

“parsing”

```
(1 + (9 – 2)) * 7
```

A Java expression as a string.




An expression as a tree.

# Applications of Tree: Syntax Trees



- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is *implicit* in ordinary textual representation
- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

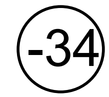
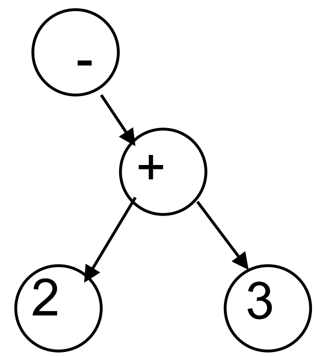
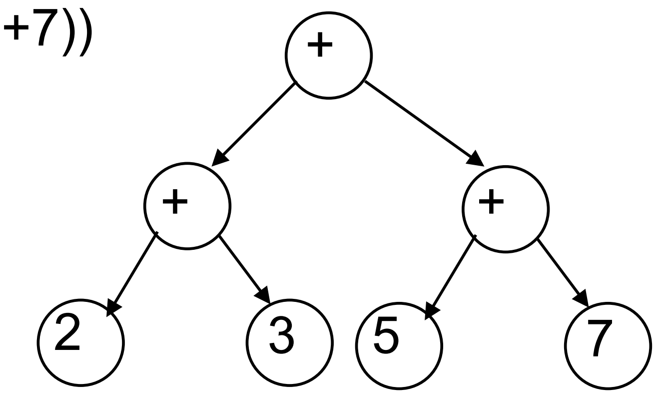
# Applications of Tree: Syntax Trees



In textual representation:
Parentheses show hierarchical structure

In tree representation:
Hierarchy is explicit in the structure of the tree

We'll talk more about expressions and trees in next lecture

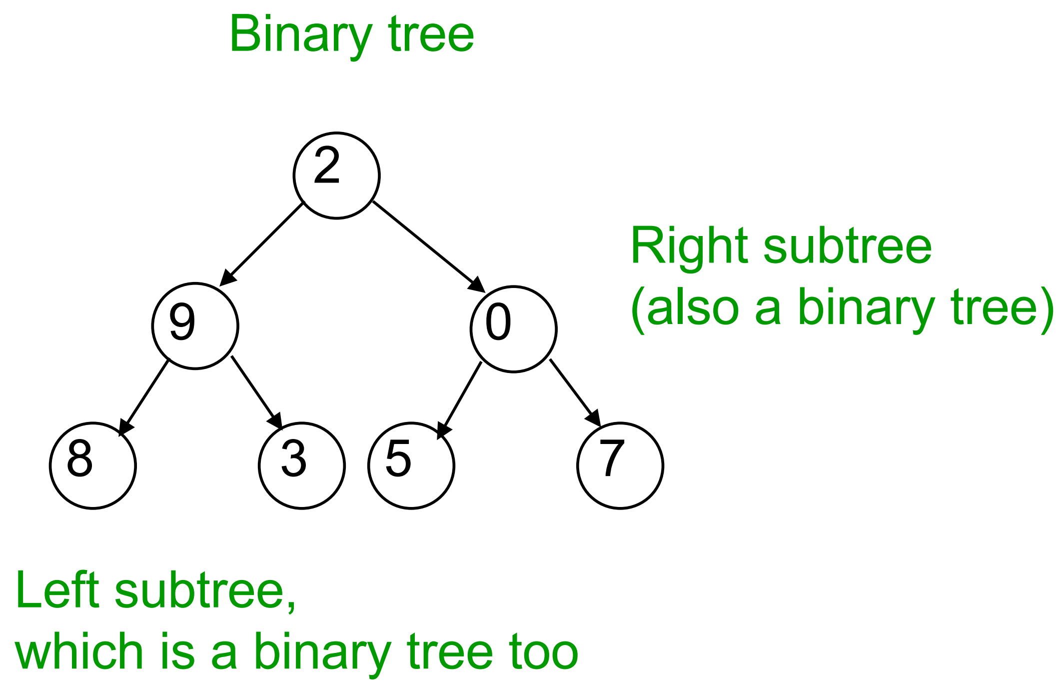
| Text | Tree Representation |
| --- | --- |
| -34 |  |
| - (2 + 3) |  |
| ((2+3) + (5+7)) |  |

# A Tree is a Recursive Thing

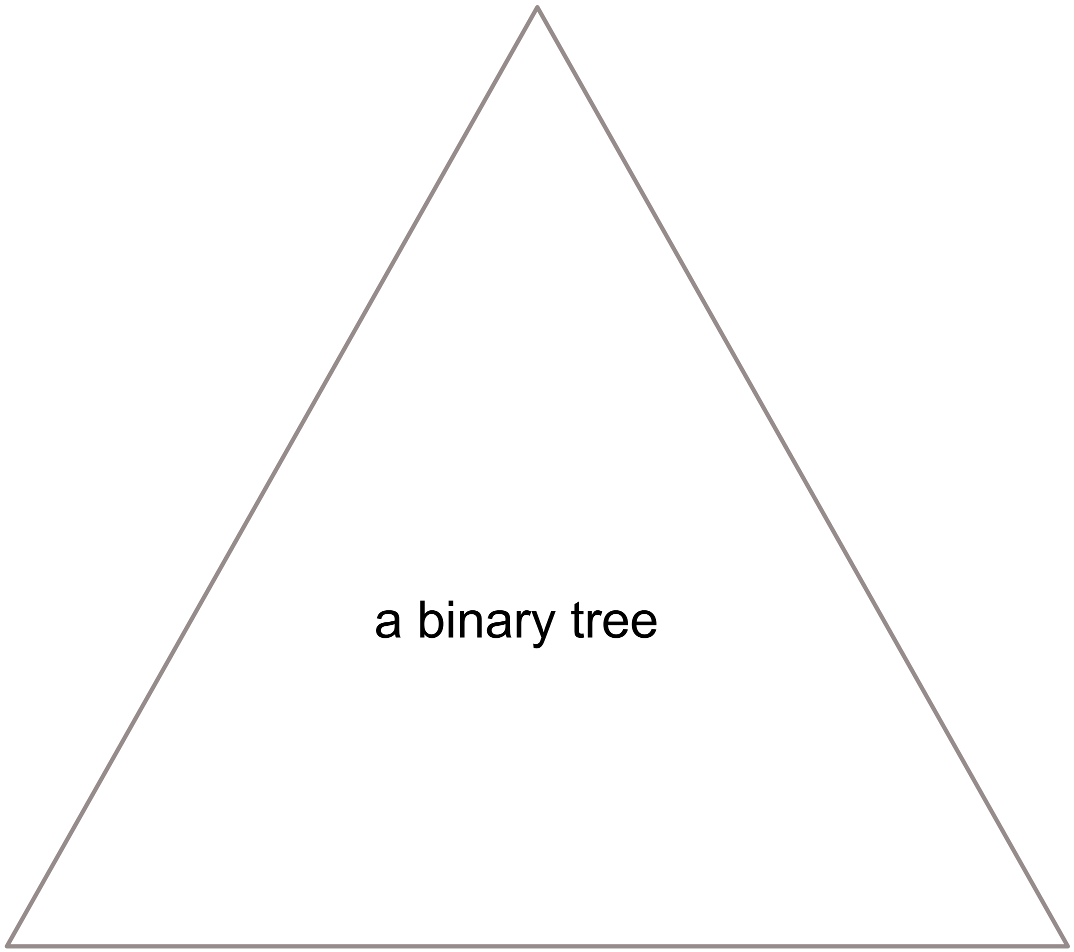


A binary tree is either `null` or an object consisting of a value, a left binary tree, and a right binary tree.

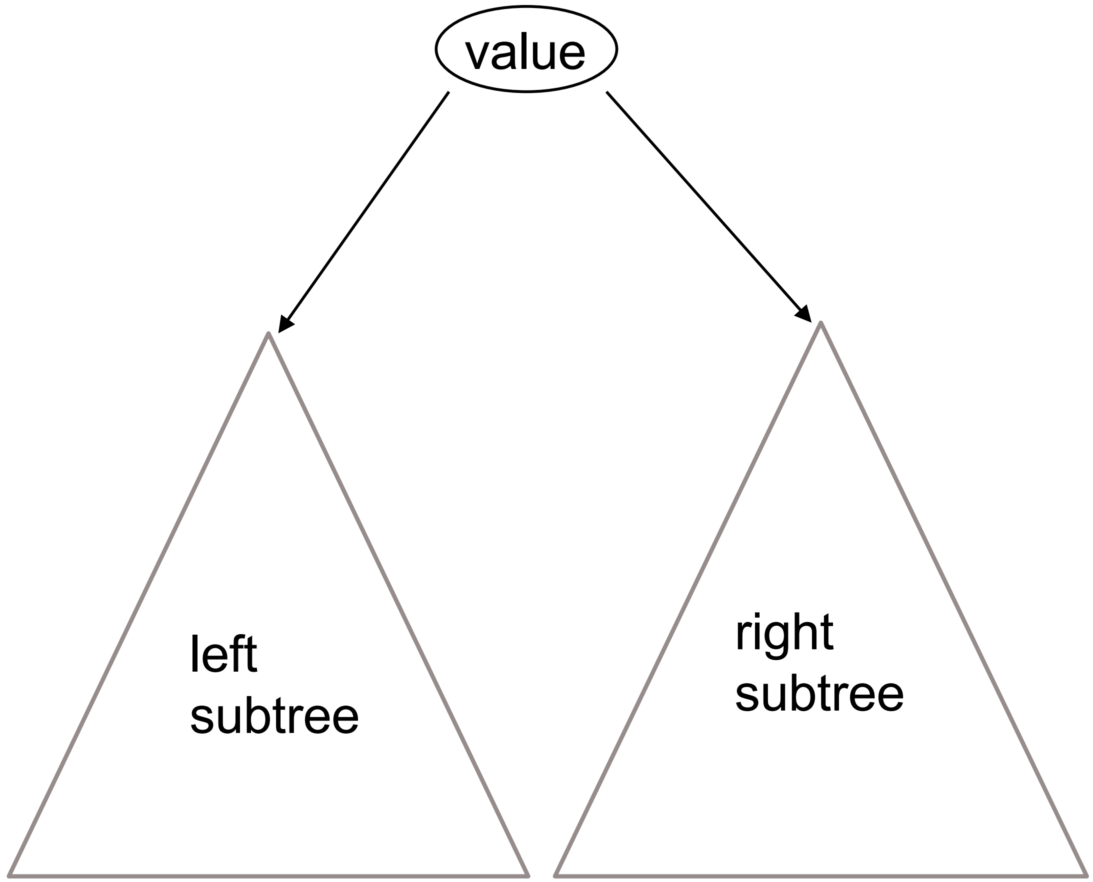
# Looking at trees recursively



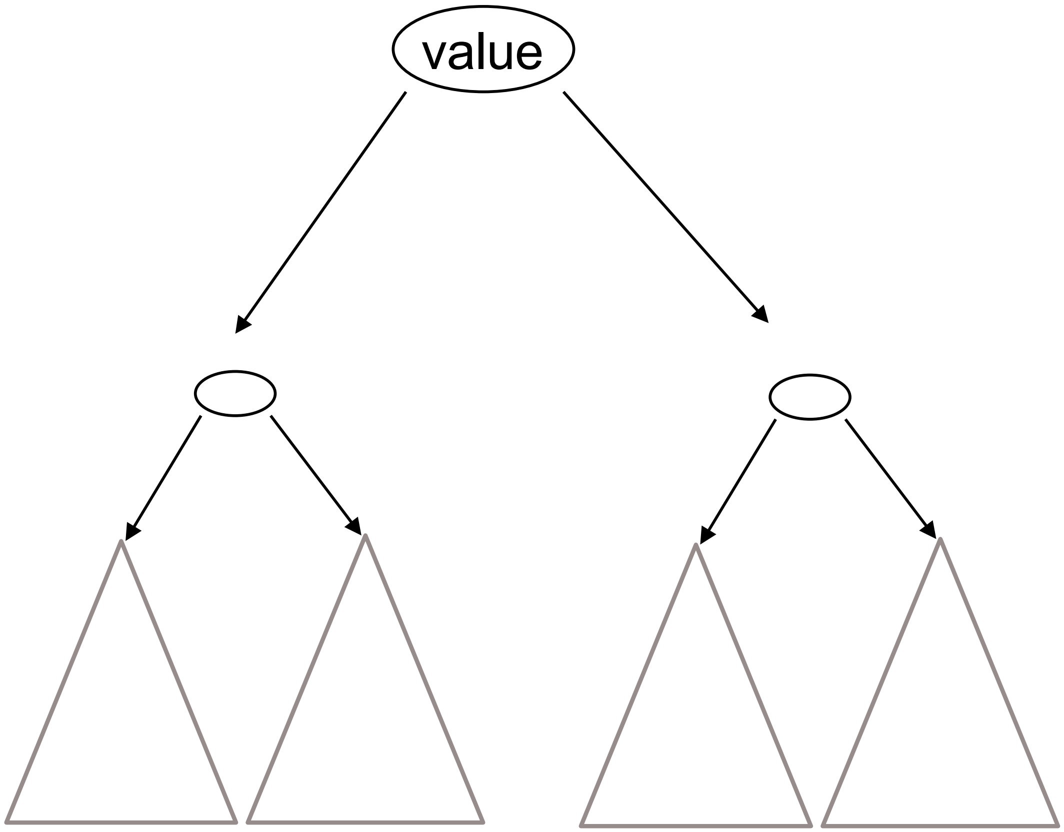
# Looking at trees recursively



# Looking at trees recursively



# Looking at trees recursively



# A Recipe for Recursive Functions



Base case:

If the input is "easy," just solve the problem directly.

Recursive case:

Get a smaller part of the input (or several parts).
Call the function on the smaller value(s).
Use the recursive result to build a solution for the full input.

# Recursive Functions on Binary Trees



Base case:

empty tree (null)

or, possibly, a leaf

Recursive case:

Call the function on each subtree.

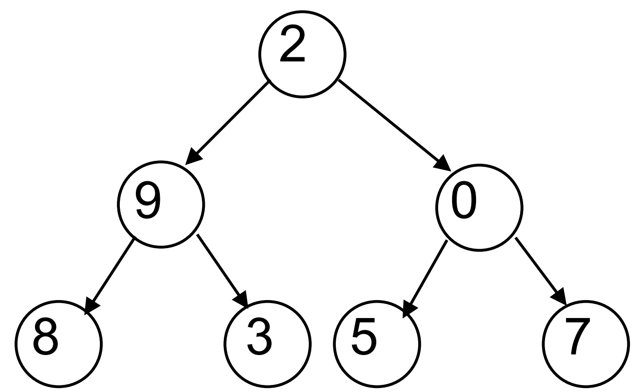
Use the recursive result to build a solution for the full input.

# Searching in a Binary Tree



```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



# Searching in a Binary Tree

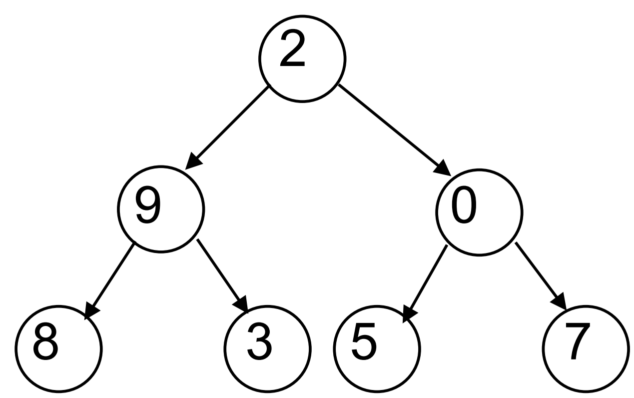


```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

VERY IMPORTANT!

We sometimes talk of t as the root of the tree.

But we also use t to denote the whole tree.



# Some useful methods – what do they do?



```
/** Method A ??? */
public static boolean A(Node n) {
   return n != null  &&  n.left == null  &&  n.right == null;
}
/** Method B ???  */
public static int B(Node n) {
   if (n== null) return -1;
   return 1 + Math.max(B(n.left), B(n.right));
}
/** Method C ???  */
public static int C(Node n) {
   if (n== null) return 0;
   return 1 + C(n.left) + C(n.right);
}
```

# Some useful methods



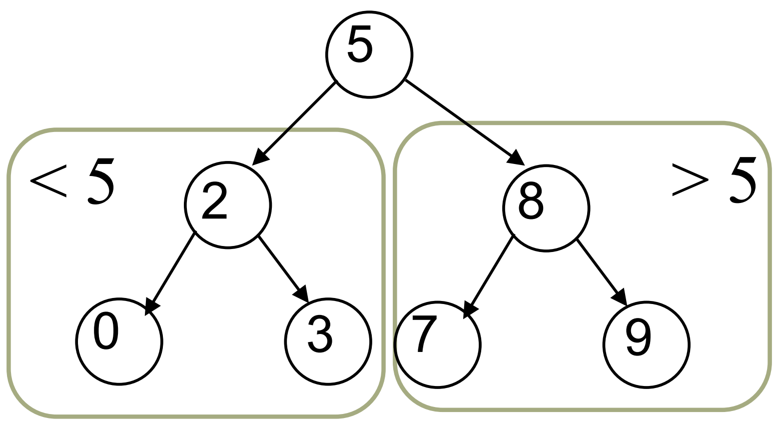
```
/** Return true iff node n is a leaf */
public static boolean isLeaf(Node n) {
   return n != null  &&  n.left == null  &&  n.right == null;
}
/** Return height of node n (postorder traversal) */
public static int height(Node n) {
   if (n== null) return -1; //empty tree
   return 1 + Math.max(height(n.left), height(n.right));
}
/** Return number of nodes in n (preorder traversal) */
public static int numNodes(Node n) {
   if (n== null) return 0;
   return 1 + numNodes(n.left) + numNodes(n.right);
}
```

# Binary Search Tree (BST)



A *binary search tree* is a binary tree that is **ordered** and **has no duplicate values.** In other words, for *every* node:

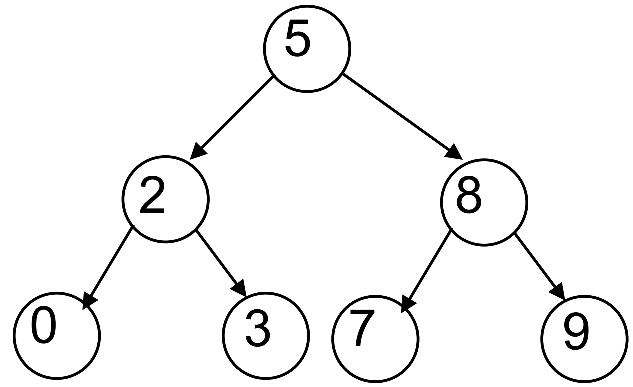
- All nodes in the left subtree have values that are less than the value in that node, and
- All values in the right subtree are greater.




A BST is the key to making search way faster.

# Binary Search Tree (BST)






Compare binary tree to binary search tree:

```
boolean searchBT(n, v):
  if n==null, return false
  if n.v == v, return true
  return searchBST(n.left, v)
      || searchBST(n.right, v)
```

2 recursive calls

```
boolean searchBST(n, v):
  if n==null, return false
  if n.v == v, return true
  if v < n.v
    return searchBST(n.left, v)
  else
    return searchBST(n.right, v)
```

1 recursive call

# Building a BST



- To insert a new item:
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree

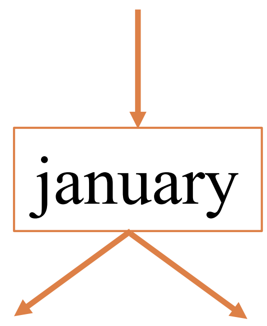
# Building a BST



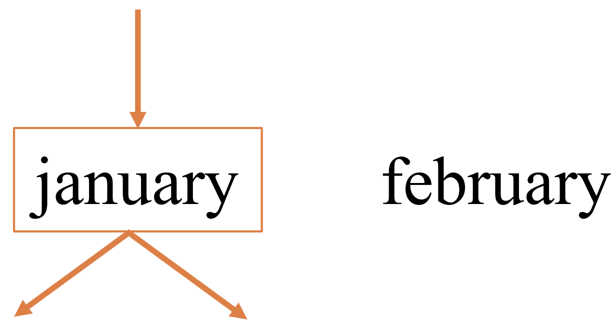
january

# Building a BST



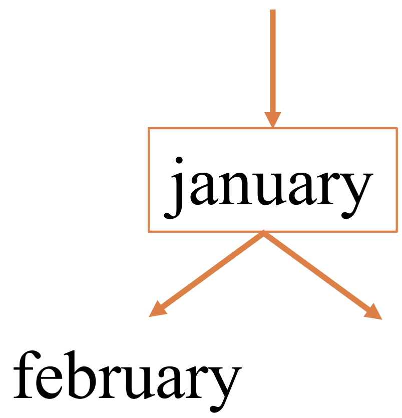


# Building a BST



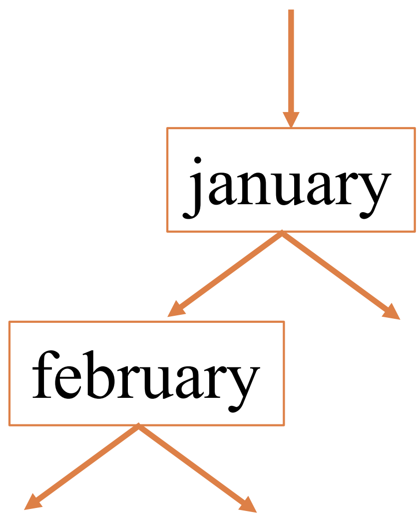
# Building a BST





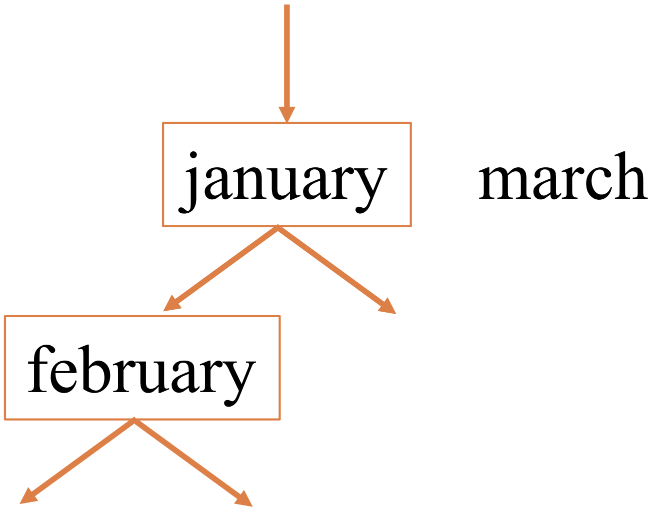
# Building a BST





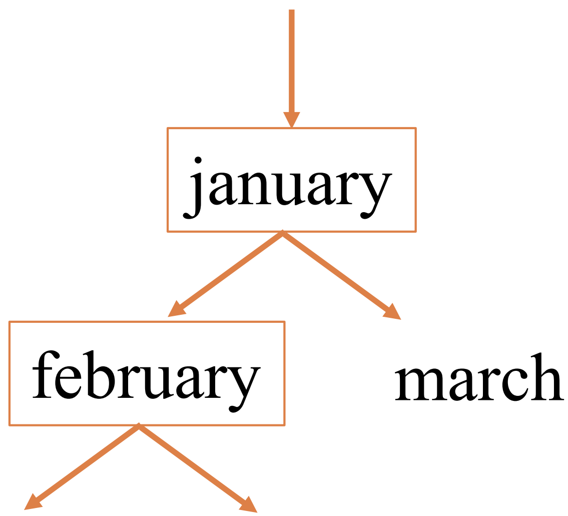
# Building a BST





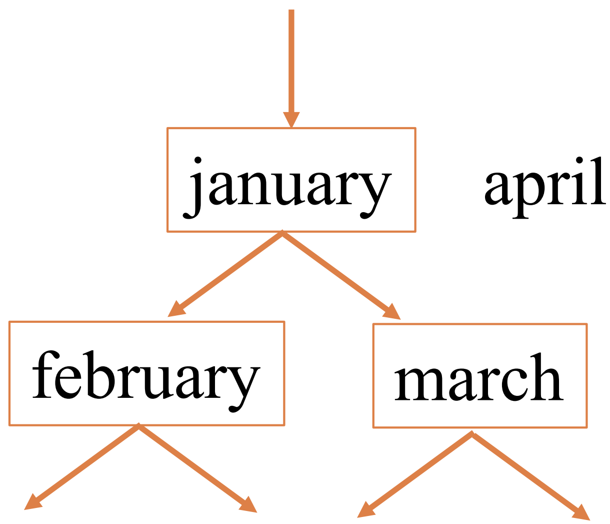
# Building a BST





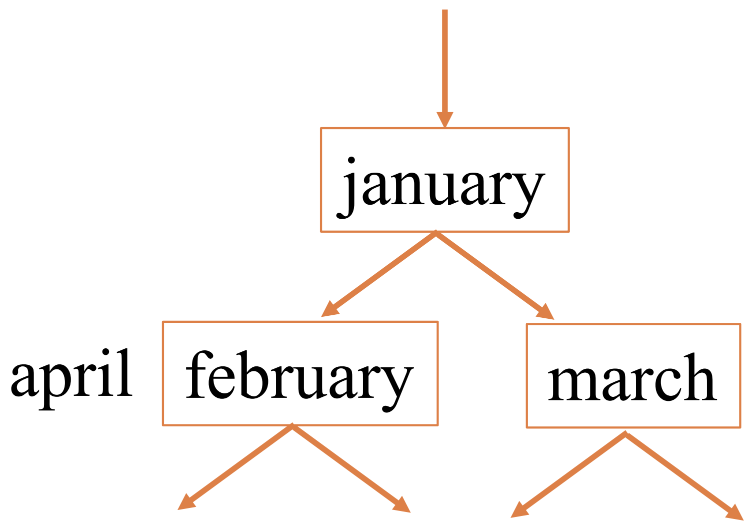
# Building a BST





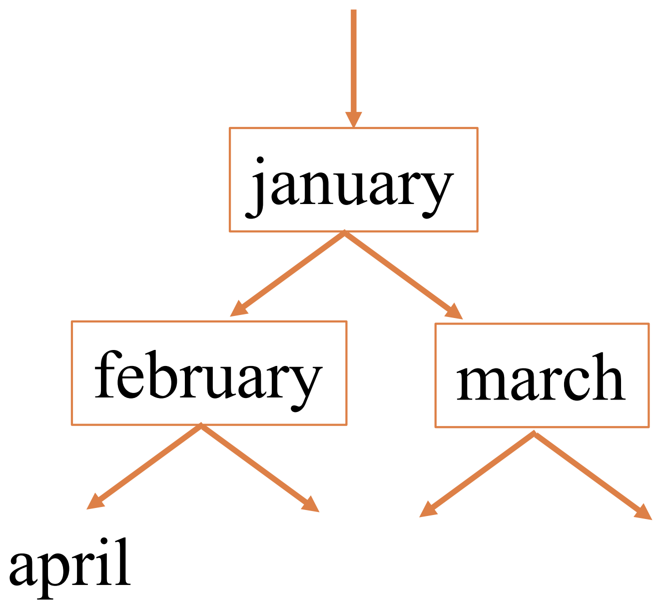
# Building a BST





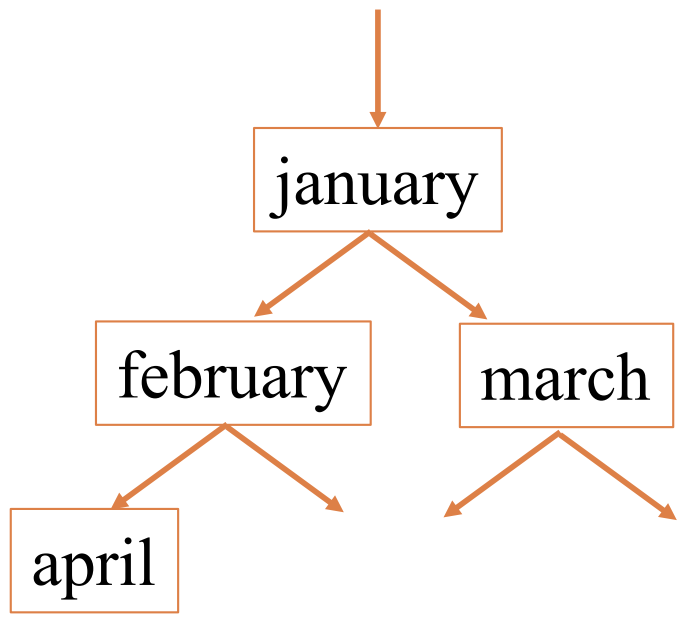
# Building a BST





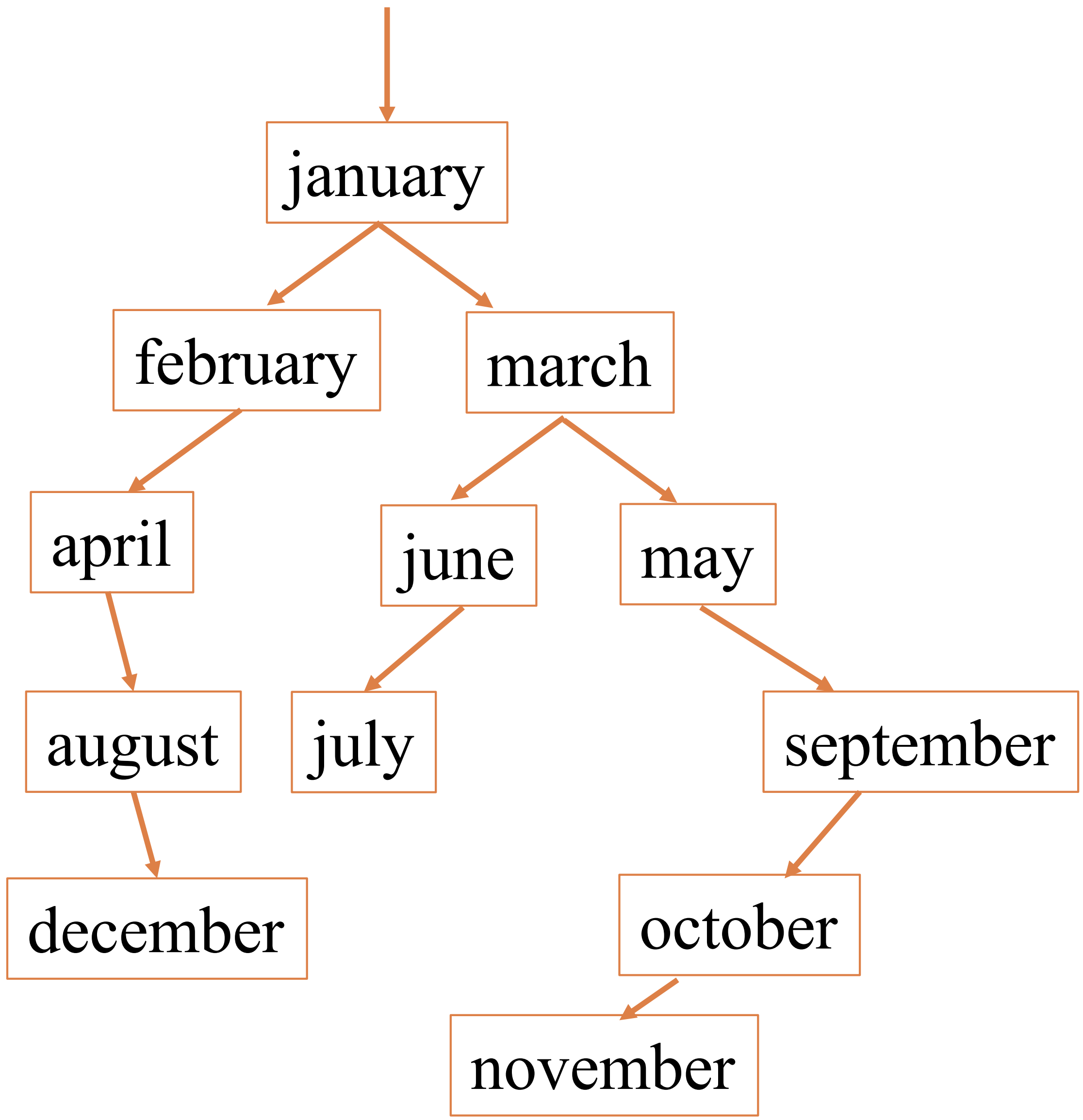
# Building a BST





# Building a BST





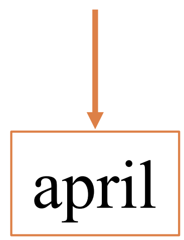
# Inserting in Alphabetical Order



april

# Inserting in Alphabetical Order





# Inserting in Alphabetical Order

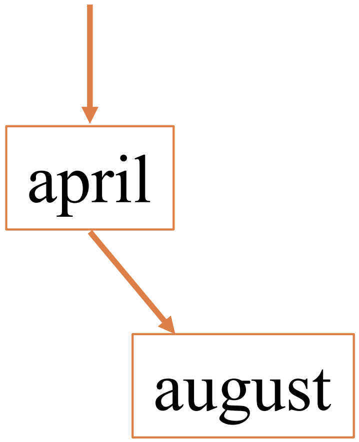


april

august

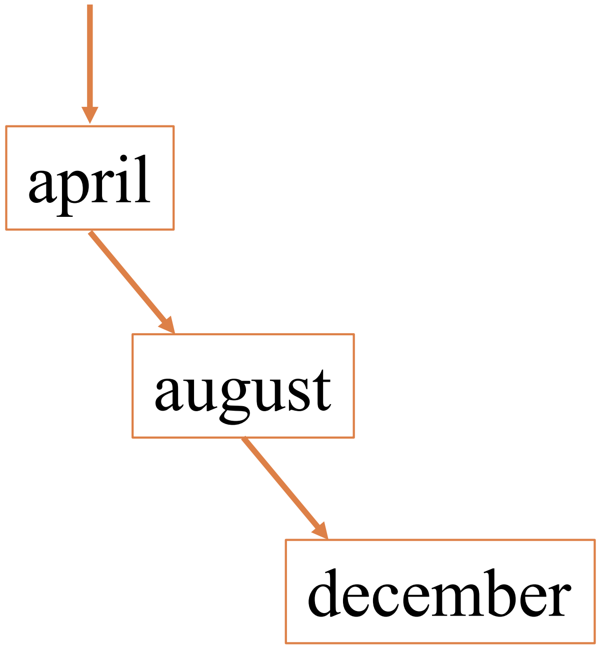
# Inserting in Alphabetical Order





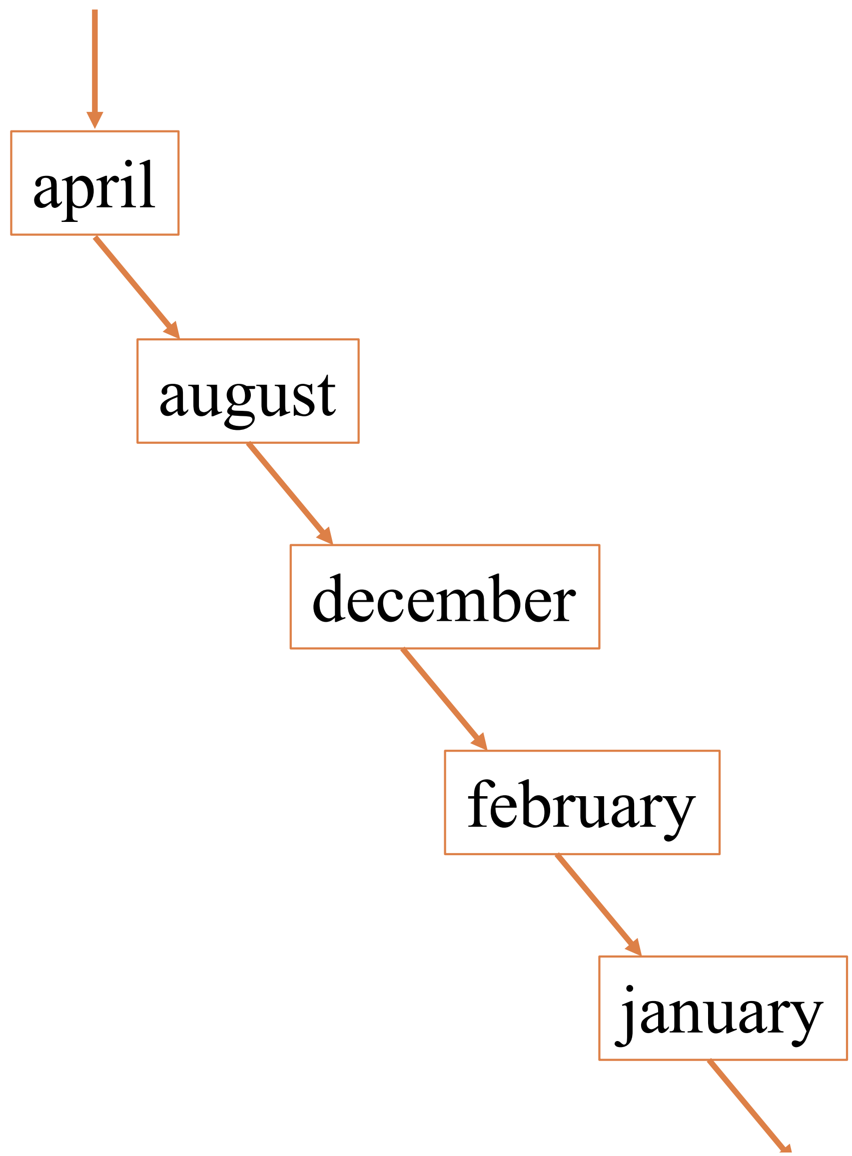
# Inserting in Alphabetical Order





# Inserting in Alphabetical Order





# Insertion Order Matters



- A *balanced* binary tree is one where the two subtrees of any node are about the same size.
- Searching a binary search tree takes O(h) time, where h is the height of the tree.
- In a balanced binary search tree, this is O(log n).
- But if you insert data in sorted order, the tree becomes imbalanced, so searching is O(n).

# Printing contents of BST



Because of ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
   if (t== null) return;
   print(t.left);
   System.out.print(t.value);
   print(t.right);
}
```

# Tree traversals



"Walking" over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example:
in-order traversal

- Process left subtree
- Process root
- Process right subtree

Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree
- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root
- level-order traversal
  - Not recursive: uses a queue (we'll cover this later)

# Useful facts about binary trees



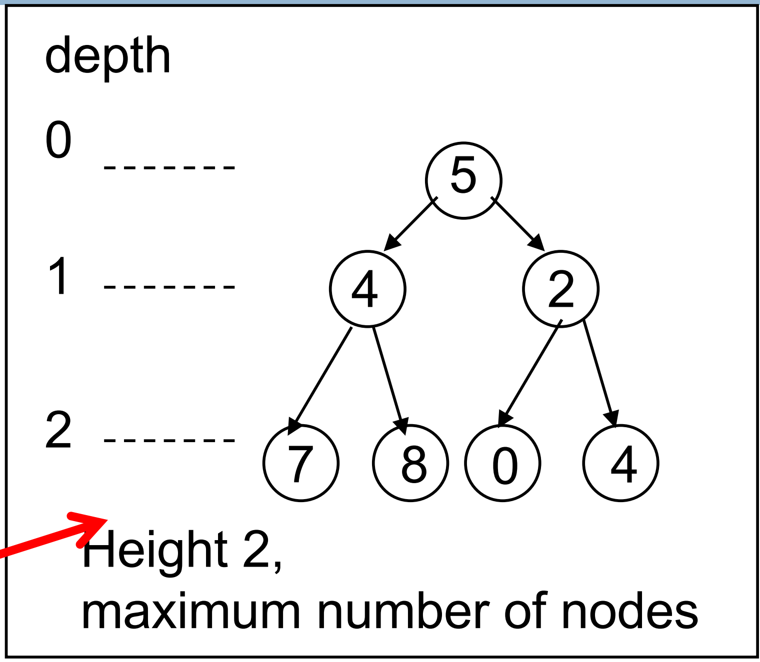
Max # of nodes at depth d: $2^d$

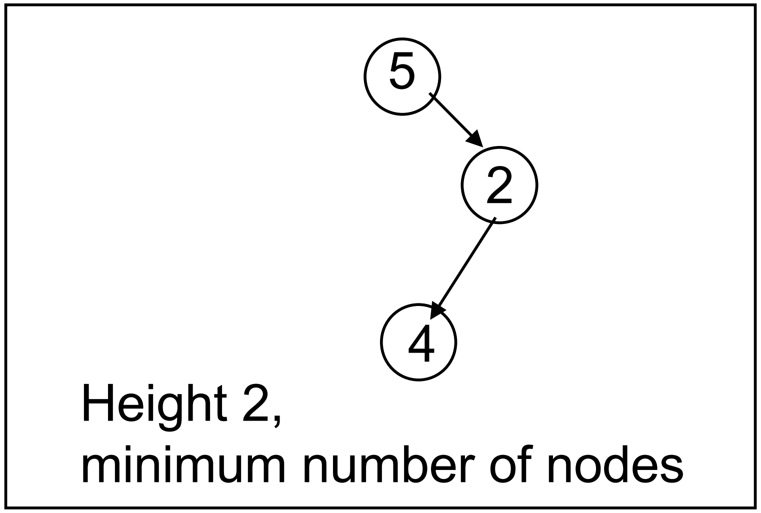
If height of tree is h

- min # of nodes: $h + 1$
- max #of nodes in tree:
- $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree

- All levels of tree down to a certain depth are completely filled

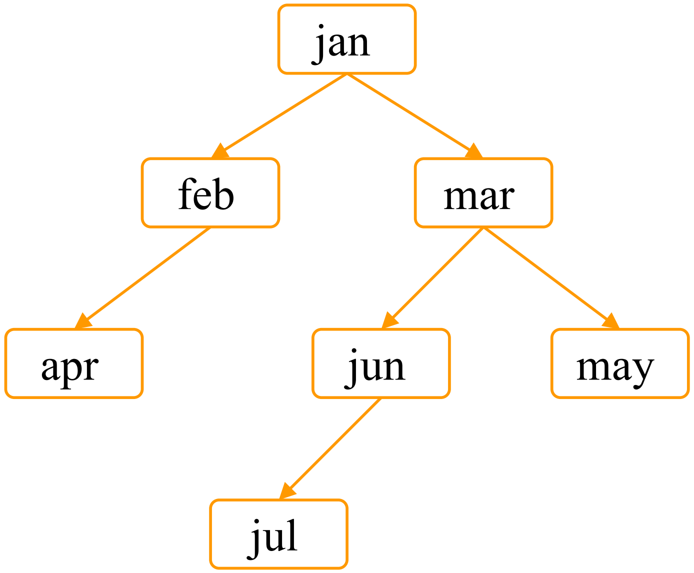


# Things to think about



What if we want to *delete* data from a BST?




A BST works great as long as it's *balanced*.

There are kinds of trees that can *automatically* keep themselves balanced as you insert things!

# Tree Summary



- A *tree* is a recursive data structure
  - Each node has 0 or more successors (*children*)
  - Each node except the *root* has exactly one predecessor (*parent*)
  - All node are reachable from the *root*
  - A node with no children (or empty children) is called a *leaf*
- Special case: *binary tree*
  - Binary tree nodes have a left and a right child
  - Either or both children can be empty (null)
- Trees are useful in many situations, including exposing the recursive structure of natural language and computer programs