TREES

Lecture 12
CS2110 – Fall 2017

Important Announcements

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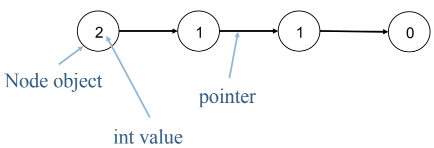
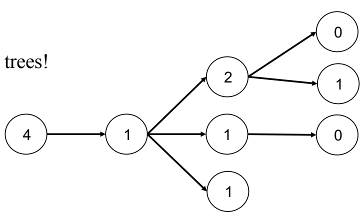
- A4 is out now and due two weeks from today. Have fun, and start early!

A picture of a singly linked list:



Today: trees!



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Tree Overview

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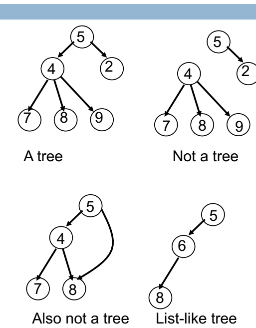
Tree: data structure with nodes, similar to linked list

- Each node may have zero or more *successors* (children)
- Each node has exactly one *predecessor* (parent) except the *root*, which has none
- All nodes are reachable from *root*



Binary Trees

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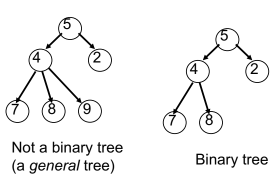
A *binary tree* is a particularly important kind of tree where every node as at most two children.

In a binary tree, the two children are called the *left* and *right* children.



Binary trees were in A1!

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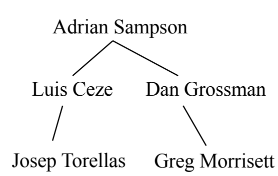
You have seen a binary tree in A1.

A PhD object has one or two advisors. (Confusingly, my advisors are my “children.”)



Tree Terminology

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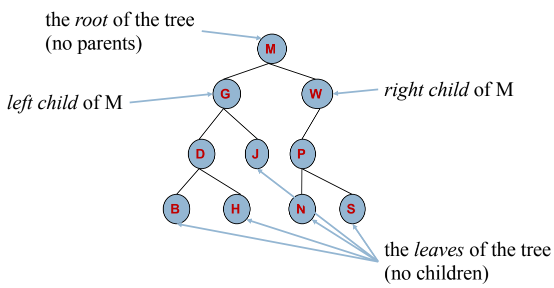

Tree Terminology

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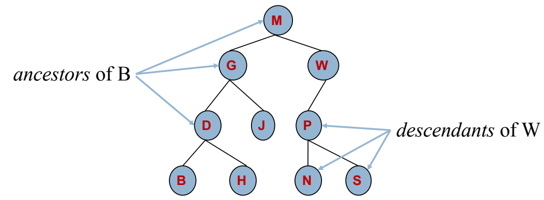

Tree Terminology

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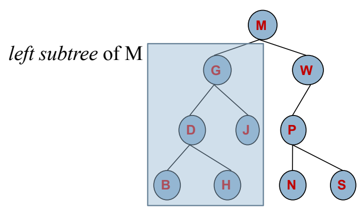

Tree Terminology

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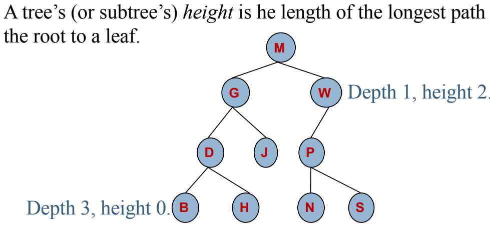
A node's *depth* is the length of the path to the root.
A tree's (or subtree's) *height* is he length of the longest path from the root to a leaf.



Tree Terminology

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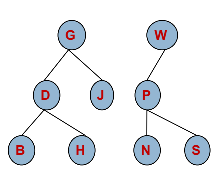
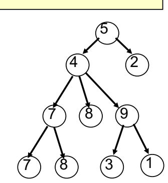
Multiple trees: a *forest*.



Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Parent contains a list of its children

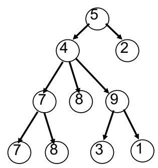


General tree

Class for general tree nodes

```
class GTreeNode<T> {
    private T value;
    private List<GTreeNode<T>> children;
    //appropriate constructors, getters,
    //setters, etc.
}
```

Java.util.List is an interface!
It defines the methods that all implementation must implement.
Whoever writes this class gets to decide what implementation to use — ArrayList? LinkedList? Etc.?



General tree

Class for binary tree node

```
class TreeNode<T> {
   private T value;
   private TreeNode<T> left, right;

   /** Constructor: one-node tree with datum x */
   public TreeNode (T d) { datum= d; left= null; right= null;}

   /** Constr: Tree with root value x, left tree l, right tree r */
   public TreeNode (T d, TreeNode<T> l, TreeNode<T> r) {
      datum= d; left= l; right= r;
   }
}
```

Either might be null if the subtree is empty.

more methods: getValue, setValue, getLeft, setLeft, etc.

Binary versus general tree

In a binary tree, each node has up to two pointers: to the left subtree and to the right subtree:

- One or both could be **null**, meaning the subtree is empty (remember, a tree is a set of nodes)

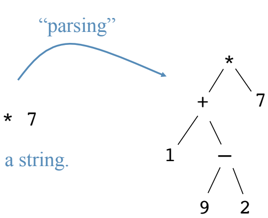
In a general tree, a node can have any number of child nodes (and they need not be ordered)

- Very useful in some situations ...
- ... one of which may be in an assignment!

An Application: Syntax Trees

`(1 + (9 – 2)) * 7`

A Java expression as a string.



An expression as a tree.

Applications of Tree: Syntax Trees

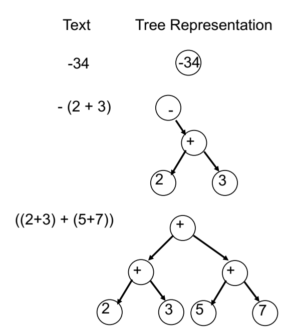
- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is *implicit* in ordinary textual representation
- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

Applications of Tree: Syntax Trees

In textual representation:
Parentheses show hierarchical structure

In tree representation:
Hierarchy is explicit in the structure of the tree

We'll talk more about expressions and trees in next lecture



A Tree is a Recursive Thing

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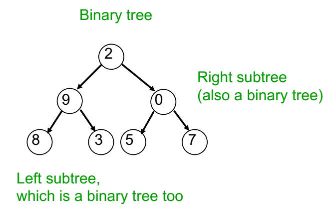
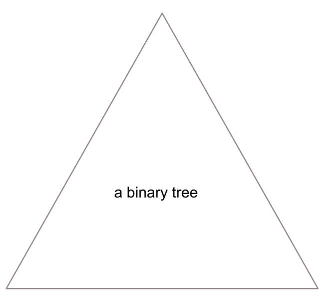
A binary tree is either `null` or an object consisting of a value, a left binary tree, and a right binary tree.

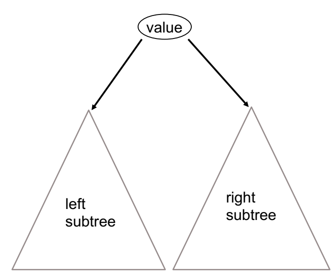
Looking at trees recursively



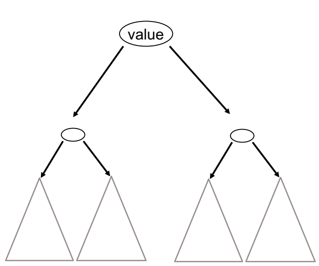
Looking at trees recursively



Looking at trees recursively



Looking at trees recursively



A Recipe for Recursive Functions

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Base case:

If the input is "easy," just solve the problem directly.

Recursive case:

Get a smaller part of the input (or several parts).
Call the function on the smaller value(s).
Use the recursive result to build a solution for the full input.

Recursive Functions on Binary Trees

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Base case:
- empty tree (null)
- or, possibly, a leaf

Recursive case:
- Call the function on each subtree.
- Use the recursive result to build a solution for the full input.

Searching in a Binary Tree

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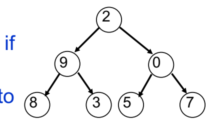
```
/** Return true iff x is the datum in a node of tree  t*/
public static boolean treeSearch(T x, TreeNode<T> t) {
   if (t == null) return false;
   if (x.equals(t.datum)) return true;
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



Searching in a Binary Tree

27

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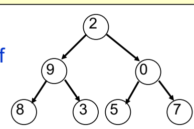
   return treeSearch(x, t.left) || treeSearch(x, t.right);
}
```

VERY IMPORTANT!

We sometimes talk of t as the root of the tree.

But we also use t to denote the whole tree.



Some useful methods – what do they do?

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```
/** Method A ??? */
public static boolean A(Node n) {
   return n != null  &&  n.left == null  &&  n.right == null;
}
/** Method B ???  */
public static int B(Node n) {
   if (n== null) return -1;
   return 1 + Math.max(B(n.left), B(n.right));
}
/** Method C ???  */
public static int C(Node n) {
   if (n== null) return 0;
   return 1 + C(n.left) + C(n.right);
}
```

Some useful methods

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```
/** Return true iff node n is a leaf */
public static boolean isLeaf(Node n) {
   return n != null  &&  n.left == null  &&  n.right == null;
}
/** Return height of node n (postorder traversal) */
public static int height(Node n) {
   if (n== null) return -1; //empty tree
   return 1 + Math.max(height(n.left), height(n.right));
}
/** Return number of nodes in n (preorder traversal) */
public static int numNodes(Node n) {
   if (n== null) return 0;
   return 1 + numNodes(n.left) + numNodes(n.right);
}
```

Binary Search Tree (BST)

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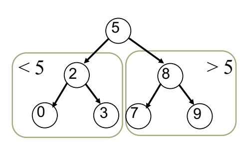
A *binary search tree* is a binary tree that is **ordered** and **has no duplicate values.** In other words, for *every* node:
- All nodes in the left subtree have values that are less than the value in that node, and
- All values in the right subtree are greater.



A BST is the key to making search way faster.

Binary Search Tree (BST)

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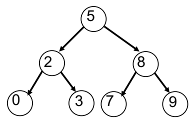

Compare binary tree to binary search tree:

```
boolean searchBT(n, v):
  if n==null, return false
  if n.v == v, return true
  return searchBST(n.left, v)
      || searchBST(n.right, v)
```

2 recursive calls

```
boolean searchBST(n, v):
  if n==null, return false
  if n.v == v, return true
  if v < n.v
    return searchBST(n.left, v)
  else
    return searchBST(n.right, v)
```

1 recursive call

Building a BST

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- To insert a new item:
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree

Building a BST

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january

Building a BST

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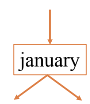

Building a BST

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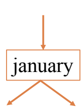

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Building a BST

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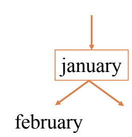

Building a BST

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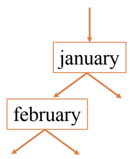

Building a BST

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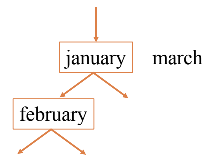

Building a BST

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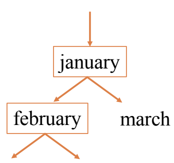
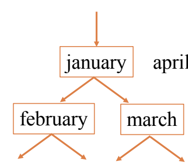


Building a BST

40



Building a BST

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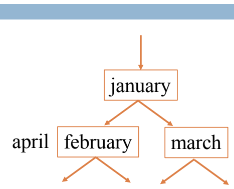

Building a BST

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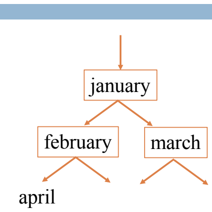
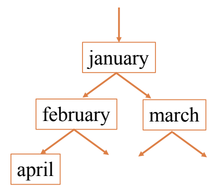
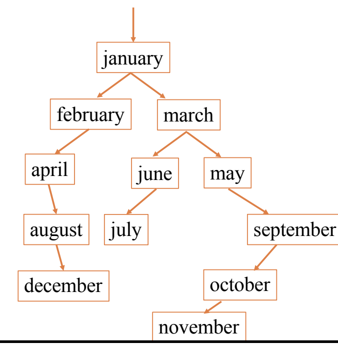


Building a BST



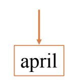
Building a BST



Inserting in Alphabetical Order

april

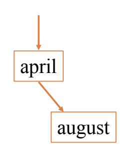
Inserting in Alphabetical Order



Inserting in Alphabetical Order

august

Inserting in Alphabetical Order



Inserting in Alphabetical Order

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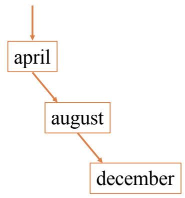

Inserting in Alphabetical Order

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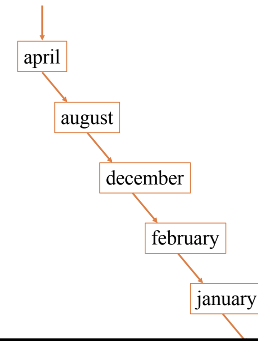

Insertion Order Matters

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- A *balanced* binary tree is one where the two subtrees of any node are about the same size.
- Searching a binary search tree takes O(h) time, where h is the height of the tree.
- In a balanced binary search tree, this is O(log n).
- But if you insert data in sorted order, the tree becomes imbalanced, so searching is O(n).

Printing contents of BST

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Because of ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print BST t in alpha order */
private static void print(TreeNode<T> t) {
   if (t== null) return;
   print(t.left);
   System.out.print(t.value);
   print(t.right);
}
```

Tree traversals

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"Walking" over the whole tree is a tree traversal

- Done often enough that there are standard names

Previous example: in-order traversal

- Process left subtree
- Process root
- Process right subtree

Note: Can do other processing besides printing

Other standard kinds of traversals

- preorder traversal
  - Process root
  - Process left subtree
  - Process right subtree
- postorder traversal
  - Process left subtree
  - Process right subtree
  - Process root
- level-order traversal
  - Not recursive: uses a queue (we'll cover this later)

Useful facts about binary trees

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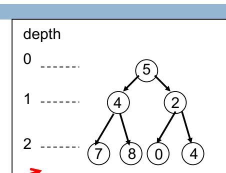
Max # of nodes at depth d: 2^d

If height of tree is h

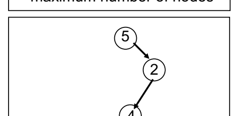
- min # of nodes: $h + 1$
- max #of nodes in tree:
- $2^0 + \ldots + 2^h = 2^{h+1} - 1$

Complete binary tree

- All levels of tree down to a certain depth are completely filled



Height 2, maximum number of nodes



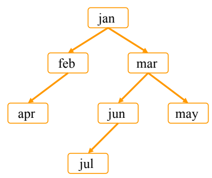
Height 2, minimum number of nodes

Things to think about

What if we want to *delete* data from a BST?

A BST works great as long as it's *balanced*.

There are kinds of trees that can *automatically* keep themselves balanced as you insert things!



Tree Summary

- A *tree* is a recursive data structure
  - Each node has 0 or more successors (*children*)
  - Each node except the *root* has exactly one predecessor (*parent*)
  - All node are reachable from the *root*
  - A node with no children (or empty children) is called a *leaf*
- Special case: *binary tree*
  - Binary tree nodes have a left and a right child
  - Either or both children can be empty (null)
- Trees are useful in many situations, including exposing the recursive structure of natural language and computer programs