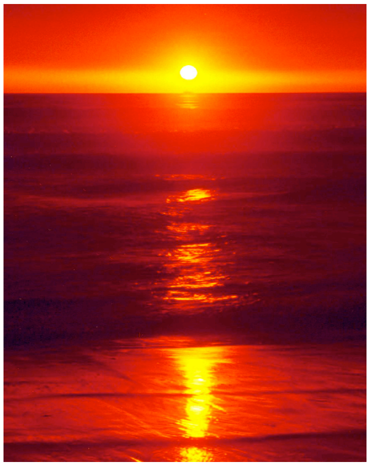


Course Review & A Few Unanswered Questions

Lecture 25
CS2110 – Fall 2008

Announcements

- Final Exam
  - Thursday, Dec 18
  - 2 - 4:30pm
  - Uris Auditorium
- Review Sessions
  - Wednesday, Dec 17
  - 7:30 - 9pm and 9 - 10:30pm
  - Upson B17
  - Both sessions the same
  - Maybe an extra one Tuesday of next week - watch web site for an announcement
- For exam conflicts:
  - Notify Kelly Patwell *today*
  - You must provide
    - Your entire exam schedule
    - Include the course numbers
- Definition of exam conflict:
  - Two exams at the same time or
  - Three or more exams within 24 hours
- A5 due Monday, Dec 8, 11:59pm
  - Sorry, no more extensions

Announcements

- Check the course website for additional announcements as the final exam approaches
  - Consulting ends this week
  - Office hours continue until Final Exam
    - There may be changes (TAs have exams, too)
    - Any changes will be announced on the course website
- Jealous of the glamorous life of a CS consultant?
  - We're recruiting next-semester consultants for CS1110 and CS2110
  - Interested students should fill out an application, available in 303 Upson

Course Evaluations

- Worth one assignment point
  - Will count as 1% of your course grade
  - This is a regular point, *not* a bonus point
  - Anonymity
    - We get a list of who completed the course evaluations and a list of responses, but no link between names & responses
- Open now
- Closes Wednesday, December 10, midnight
- http://www.engineering.cornell.edu/CourseEval
  - This link also appears on the CS2110 announcements page

Course Overview

- Programming concepts
  - We use Java, but the goal is to understand the ideas rather than to become a Java expert
  - Recursion
  - Object-Oriented Programming
  - Interfaces
  - Graphical User Interfaces (GUIs)
- Data structure concepts
  - The goal here is to develop skill with a set of tools that are widely useful
  - Induction
  - Asymptotic analysis (big-O)
  - Arrays, Trees, and Lists
  - Searching & Sorting
  - Stacks & Queues
  - Priority Queues
  - Sets & Dictionaries
  - Graphs

Programming Concepts

- Recursion
  - Stack frames
  - Exceptions
- Object-oriented programming
  - Classes and objects
  - Primitive vs. reference types
  - Dynamic vs. static types
  - Subtypes and Inheritance
    - Overriding
    - Shadowing
    - Overloading
    - Upcasting & downcasting
  - Inner & anonymous classes
- Interfaces
  - Type hierarchy vs. class hierarchy
  - The Comparable interface
  - Iterators & Iterable
- GUIs
  - Components, Containers, & Layout Managers
  - Events & listeners

Data Structure Concepts

- Induction
- Grammars & parsing
- Asymptotic analysis (big-O)
  - Solving recurrences
  - Lower bounds on sorting
- Basic building blocks
  - Arrays
  - Lists
    - Singly- and doubly-linked
  - Trees
    - Binary Search Trees (BSTs)
- Searching
  - Linear- vs. binary-search
- Sorting
  - Insertion-, Selection-, Merge-, Quick-, and Heapsort
- Useful ADTs (& implementations)
  - Stacks & Queues
    - Arrays & lists
  - Priority Queues
    - Heaps
    - Array of queues
  - Sets & Dictionaries
    - Bit vectors (for Sets)
    - Arrays & lists
    - Hashing & Hashtables
    - BSTs (& balanced BSTs)
  - Graphs...

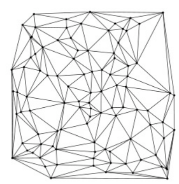
Overview of Graphs

- Mathematical definition of a graph (directed, undirected)
- Representations
  - Adjacency matrix
  - Adjacency list
- Topological sort
- Coloring & planarity
- Searching (BFS & DFS)
- Dijkstra's shortest path algorithm
- Minimum Spanning Trees (MSTs)
  - Prim's algorithm (growing a single tree)
  - Kruskal's algorithm (build a forest by adding edges in order)

Some Unsolved Problems

Complexity of Bounded-Degree Euclidean MST

- The Euclidean MST (Minimum Spanning Tree) problem:
  - Given n points in the plane, determine the MST
  - Can be solved in O(n log n) time by first building the Delaunay Triangulation



- Bounded-degree version:
  - Given n points in the plane, determine a MST where each vertex has degree ≤ d
    - Known to be NP-hard for d=3 [Papadimitriou & Vazirani 84]
    - O(n log n) algorithm for d=5 or greater
      - Can show Euclidean MST has degree ≤ 5
    - Unknown for d=4

Complexity of Euclidean MST in R^d

- Given n points in dimension d, determine the MST
  - Is there an algorithm with runtime close to the $\Omega(n \log n)$ lower bound?
- Can solve in time O(n log n) for d=2
- For large d, it appears that runtime approaches $O(n^2)$
- Best algorithms for general graphs run in time linear in m = number of edges
  - But for Euclidean distances on points, the number of edges is n(n-1)/2

$O(n^2)$ Time for X+Y Sorting?

How long does it take to a sort an n-by-n table of numbers?

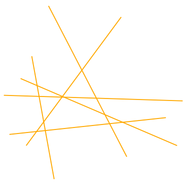
n-by-n

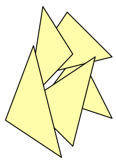
+	1	3	5	8
2	3	5	7	10
10	11	13	15	18
12	13	15	17	20
14	15	17	19	22

- $O(n^2 \log n)$ because there are n^2 numbers in the table
- What if it's an *addition table*?
  - Shouldn't it be easier to sort than an arbitrary set of n^2 numbers?
- There is a technique that uses just $O(n^2)$ *comparisons* [Fredman 76]
  - But it uses $O(n^2 \log n)$ time to decide *which* comparisons to use [Lambert 92]
- This problem is closely related to the problem of sorting the vertices of a line arrangement

3SUM in Subquadratic Time?

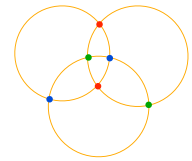
- Given a set of n integers, are there three that sum to zero?
  - $O(n^2)$ algorithms are easy (e.g., use a hashtable)
  - Are there better algorithms?
- This problem is closely related to many other problems [Gajentaan & Overmars 95]
  - Given n lines in the plane, are there 3 lines that intersect in a point?
  - Given n triangles in the plane, does their union have a hole?





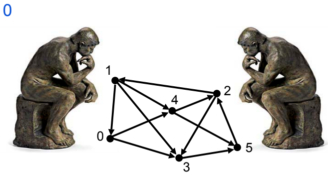
3-Colorability of Great-Circle Graphs?

- Build a graph by drawing great-circles on a sphere
  - Create a vertex for each intersection
  - Assume no three great circles intersect in a point
- Is the resulting graph 3-colorable?
- All arrangements for up to 11 great circles have been verified as 3-colorable
- For *general* circles on the sphere (or for circles on the plane) the graph can require 4 colors



Winning Strategies for the Parity Game?

- Played on a directed graph with nodes 0, 1, 2, ..., n−1
- Start with a pebble on node 0
- Players Steven and Todd alternately choose edges along which to push the pebble
- They play forever...
- Steven wins if the least-numbered vertex visited infinitely often is even



- Todd wins if the least-numbered vertex visited infinitely often is odd
- It is known that for any graph, either Steven or Todd has a winning strategy − but can you determine *which*?
- Equivalent to a major open problem in logic

The Big Question: Is P=NP?

- P is the class of problems that can be solved in polynomial time
  - These problems are considered *tractable*
  - Problems that are not in P are considered *intractable*
- NP represents problems that, for a *given solution*, the solution can be *checked* in polynomial time
  - But *finding* the solution may be hard
- For ease of comparison, problems are usually stated as yes-or-no questions
- Examples
  - Given a weighted graph G and a bound k, does G have a spanning tree of weight at most k?
    - This is in P because we have an algorithm for the MST with runtime O(m + n log n)
  - Given graph G, does G have a Hamiltonian cycle (a simple cycle that visits all vertices)?
    - This is in NP because, given a possible solution, we can check in polynomial time that it's a cycle and that it visits all vertices exactly once

Current Status: P vs. NP

- It's easy to show that $P \subseteq NP$
- Most researchers believe that $P \neq NP$
  - But at present, no proof
  - We do have a large collection of *NP-complete problems*
    - If *any* NP-complete problem has a polynomial time algorithm, then they *all* do
- A problem B is *NP-complete* if
  1. it is in NP
  2. any other problem in NP reduces to it efficiently
- Thus by making use of an *imaginary* fast subroutine for B, any problem in NP could be solved in polynomial time
  - the Boolean satisfiability problem is NP-complete [Cook 1971]
  - many useful problems are NP-complete [Karp 1972]
  - By now thousands of problems are known to be NP-complete

Some NP-Complete Problems

- Graph coloring: Given graph G and bound k, is G k-colorable?
- Planar 3-coloring: Given planar graph G, is G 3-colorable?
- Traveling salesperson: Given weighted graph G and bound k, is there a cycle of cost ≤ k that visits each vertex at least once?
- Hamiltonian cycle: Give graph G, is there a cycle that visits each vertex exactly once?
- Knapsack: Given a set of items i with weights w_i and values v_i, and numbers W and V, does there exist a subset of at most W items whose total value is at least V?
- What if you really *need* an algorithm for an NP-complete problem?
  - Some special cases can be solved in polynomial time
    - If you're lucky, you have such a special case
  - Otherwise, once a problem is shown to be NP-complete, the best strategy is to start looking for an approximation
- For a while, a new proof showing a problem NP-complete was enough for a paper
  - Nowadays, no one is interested unless the result is somehow unexpected

Good luck on the final!

Thanks for an enjoyable semester!

Have a great winter break!

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