Priority Queues and Heaps

CS211
Fall 2000

ADT Priority Queue

- Operations:
  - boolean isEmpty();
  - void add (Object item);
  - Object removeFirst ();
- Other less-common operations:
  - update (an Item's priority)
  - join two PQs to make one new PQ
  - delete (an Item)
- Uses
  - Job scheduler for OS
  - Can use to sort
  - Retain the best k items
  - Event-driven simulation
  - Wide use within other algorithms

Possible PQ Implementations

	Unordered List	Ordered List	Unordered Array	Ordered Array	BST*	Balanced BST
add(item)	O(1)	O(n)	O(1)	O(n)	O(log n) expected	O(log n) worst-case
removeFirst()	O(n)	O(1)	O(n)	O(1)	O(log n) expected	O(log n) worst-case

* BST becomes unbalanced as PQ is used

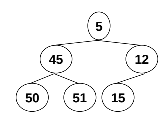
Can we do better than balanced trees?
Well no, not in terms of big-O bounds, but…

Heaps

<u>Definition</u>: A *min-heap* is a complete binary tree in which the value at each node is ≤ the value of its children

<u>Definition</u>: For a *max-heap*, each node is ≥ the value of its children

<u>Definition</u>: *Complete* means that each level of the tree is filled except possibly the last, which is filled from left to right



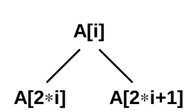
A Min-Heap

Add and RemoveFirst (for min-heap)

```
add (Item):
    Place item in next empty
        position;
    while (item < parent) {
        Swap item with parent;
    }
```

```
removeFirst ():
    min = root.value;
    Swap root and last item in heap;
    Decrease heap size by 1;
    // The last item (call it v) is at root.
    while (v > one of its children) {
        Swap v with its smallest child;
        }
    return min;
```


Heap Implementation (the Big Trick)



- Can avoid using pointers!
- Store the heap in an array
- For A[i]
  left child = 2 ∗ i
  right child = 2 ∗ i + 1
  parent = ⌊i / 2⌋

To Build a Heap

- How long to construct a heap, given the items?
- Worst-case time for insert() is O(log n)
- Total time to build heap using insert() is
  O(log 1) + O(log 2) + ... + O(log n)
  or O(n log n)

Can we do better?

- We had two heap-fixing methods
  - bubbleUp: move up the tree as long as we're less than our parent
  - bubbleDown: move down the tree as long as we're bigger than one of our children
- If we build the heap from the bottom-up using bubbleDown then we can build it in time O(n) (Wow!)

Efficient Heap Building

- Build from the bottom-up
- If there are n items in the heap then...
  - There are about n/2 mini-heaps of height 1
  - There are about n/4 mini-heaps of height 2
  - There are about n/8 mini-heaps of height 3 and so on
- The time to fix up a mini-heap is O(its height)
- Total time spent fixing heaps is thus bounded by
  $n/2 + 2n/4 + 3n/8 + ...$
- This can be rewritten as
  $n(1/2 + 2/4 + ... + i/2^i + ...)$
  $= n(2)$
- Thus total heap-building time (using the bottom-up method) is O(n)

Other Heap Operations

delete
a particular item

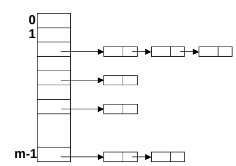
update
an item (change its priority)

join
two priority queues

- For delete and update, we need to be able to find the item
  - One way to do this: Use a HashMap to keep track of the item's position in the heap
- Efficient joining of 2 Priority Queues requires another data structure
  - Skew Heaps or Pairing Heaps (Chapter 22 in text)

Another PQ Implementation

- If there are only a few possible priorities then can use an array of lists
  - Each array position represents a priority (0..m-1 where m is the array size)
  - Each list holds all items that have that priority (treated as a queue)
- One text [Skiena] calls this a *bounded height priority queue*
- Time for add: O(1)
- Time for removeFirst:
  - O(m) in the worst-case
  - Generally, faster



PQ Application: Simulation

- Example: Given a probabilistic model of bank-customer arrival times and transaction times, how many tellers are needed
  - Assume we have a way to generate random inter-arrival times
  - Assume we have a way to generate transaction times
  - Can simulate the bank to get some idea of how long customers must wait

Time-Driven Simulation
- Check at each *tick* to see if any event occurs

Event-Driven Simulation
- Advance clock to next event, skipping intervening *ticks*
- This uses a PQ!