

CS1110 3 November 2011
insertion sort, selection sort, quick sort

Do exercises on pp. 311-312 to get familiar with concepts and develop skill. Practice in DrJava! Test your methods!

Think about the algorithm and write the invariant for it.

- Let the pre/post conditions inspire an invariant, and Let the three of them give you the algorithm.

Have a problem in your life?
 Yes ↓ Can you do something about it?
 No → Then don't worry
 Yes ↑
 No ↑

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Comments on A5

Recursion:
 Make requirements/descriptions less ambiguous, clearer; give more direction.
 Need optional problem with more complicated recursive solution would have been an interesting challenge, more recursive functions. They make us think!
 Make task 5 easier. I could not finish it.

Great! No test cases!
 Needed too much help, took too long
 Add more methods; it did not take long

Allow us to do recursive methods with loops rather than recursively.
 Go over nested loops, because some people find the concept difficult.

I had intended here to erupt in largely incoherent rage over that wretched concept of recursion, which I came to hate like an enemy: like a sentient being who, knowing the difference between right and wrong, had purposely chosen to do me harm. However, I then figured out how it works, and it is actually quite elegant, so now I suppose I have learned something against my will.

Binary search –like searching in a telephone book

Precondition P: $b[h..k]$ is in ascending order (sorted). v is some value.

Store in i to truthify
Postcondition Q: $b[h..i] \leq v$ and $v < b[i+1..k]$ Finds rightmost occurrence of v , if v in $b[h..k]$

P: b h $?$ k

Q: b h i k
 $\leq v$ $> v$

if v is 3, set i to 4
 if v is 4, set i to 6
 if v is 5, set i to 6
 if v is 8, set i to 9

Example b h 0 1 2 3 4 5 6 7 8 9
 3 3 3 3 4 4 6 7 7

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Binary search

P: b h $?$ k

Q: b h i k
 $\leq v$ $> v$

inv: b h i t k
 $\leq v$ $?$ $> v$

$i = h - 1; t = k + 1;$
while ($i + 1 \neq t$) {

Looking at $b[i+1]$ gives linear search from left.
 Looking at $b[t-1]$ gives linear search from right.
 Looking at middle: $b[(i+t)/2]$ gives a telephone-like search

}

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Binary search

P: b h $?$ k

Q: b h i k
 $\leq v$ $> v$

inv: b h i t k
 $\leq v$ $?$ $> v$

$i = h - 1; t = k + 1;$
while ($i + 1 \neq t$) {
int $e = (i + t) / 2;$
 // $\{i < e < t\}$
if ($b[e] \leq v$) $i = e;$ // makes progress, keeps inv true
else $t = e;$ // makes progress, keeps inv true
}

For $k+1-h = 2^{15} = 32768$
 Takes only 16 loop iterations!

b h i e t k
 $\leq v$ $?$ $> v$

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Sorting: "sorted" means in ascending order

pre: b 0 $?$ n post: b 0 sorted n

insertion sort 0 i n
inv: b sorted $?$

for (**int** $i = 0; i < n; i = i + 1$) {
 Push $b[i]$ down into its sorted position in $b[0..i];$
}

Iteration i makes up to i swaps.
 In worst case, number of swaps needed is $0 + 1 + 2 + 3 + \dots + (n-1) = (n-1)n / 2.$
 Called an "n-squared", or n^2 , algorithm.

$b[0..i-1]$: i elements
 in worst case:
 Iteration 0: 0 swaps
 Iteration 1: 1 swap
 Iteration 2: 2 swaps
 ...

0 i
 2 4 4 6 6 7 5
 0 i
 2 4 4 5 6 6 7

pre: b $\begin{array}{|c|c|c|} \hline 0 & ? & n \\ \hline \end{array}$ post: b $\begin{array}{|c|c|c|} \hline 0 & \text{sorted} & n \\ \hline \end{array}$

insertion sort invariant: b $\begin{array}{|c|c|c|} \hline 0 & \text{sorted} & i \\ \hline \end{array}$ $\begin{array}{|c|c|c|} \hline ? & & n \\ \hline \end{array}$

Add property to invariant: first segment contains smaller values.

selection sort invariant: b $\begin{array}{|c|c|c|} \hline 0 & \leq b[i..], \text{ sorted} & i \\ \hline \end{array}$ $\begin{array}{|c|c|c|} \hline \geq b[0..i-1], ? & & n \\ \hline \end{array}$

for (int i=0; i < n; i=i+1) {
 int j= index of min of b[i..n-1];
 Swap b[j] and b[i];
}

$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 2 & 4 & 4 & 6 & 6 & 8 & 9 & 9 & 7 & 8 & 9 \\ \hline \end{array}$

$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 2 & 4 & 4 & 6 & 6 & 7 & 9 & 9 & 8 & 8 & 9 \\ \hline \end{array}$

Also an “n-squared”, or n^2 , algorithm. 7

Partition algorithm: Given an array b[h..k] with some value x in b[h]:

P: b $\begin{array}{|c|c|c|} \hline h & ? & k \\ \hline \end{array}$

Swap elements of b[h..k] and store in j to truthify P:

Q: b $\begin{array}{|c|c|c|} \hline h & j & k \\ \hline \end{array}$
 $\begin{array}{|c|c|c|} \hline \leq x & x & \geq x \\ \hline \end{array}$

change: b $\begin{array}{|c|c|c|c|c|c|c|c|} \hline h & & & & & & & k \\ \hline 3 & 5 & 4 & 1 & 6 & 2 & 3 & 8 & 1 \\ \hline \end{array}$

into b $\begin{array}{|c|c|c|c|c|c|c|c|} \hline h & j & & & & & & k \\ \hline 1 & 2 & 1 & 3 & 5 & 4 & 6 & 3 & 8 \\ \hline \end{array}$

or b $\begin{array}{|c|c|c|c|c|c|c|c|} \hline h & j & & & & & & k \\ \hline 1 & 2 & 3 & 1 & 3 & 4 & 5 & 6 & 8 \\ \hline \end{array}$

x is called the **pivot value**.
x is not a program variable; x just denotes the value initially in b[h]. 8

Quicksort

```

/** Sort b[h..k] */
public static void qsort(int[] b, int h, int k) {
    if (b[h..k] has fewer than 2 elements)
        return;
    int j= partition(b, h, k);
    // b[h..j-1] <= b[j] <= b[j+1..k]
    // Sort b[h..j-1] and b[j+1..k]
    qsort(b, h, j-1);
    qsort(b, j+1, k);
}

```


To sort array of size n. e.g. 2^{15}
Worst case: n^2 e.g. 2^{30}
Average case: $n \log n$. e.g. $15 * 2^{15}$
 $2^{15} = 32768$

pre: b $\begin{array}{|c|c|c|} \hline h & ? & k \\ \hline \end{array}$


j= partition(b, h, k);

post: b $\begin{array}{|c|c|c|} \hline h & j & k \\ \hline \leq x & x & \geq x \\ \hline \end{array}$ 9

Tony Hoare, in 1968
Quicksort author



Tony Hoare in 2007 in Germany



Thought of Quicksort in ~1958. Tried to explain it to a colleague, but couldn't.
Few months later: he saw a draft of the definition of the language Algol 58 –later turned into Algol 60. It had recursion.
He went and explained Quicksort to his colleague, using recursion, who now understood it. 10

The NATO Software Engineering Conferences
homepages.cs.ncl.ac.uk/brian.randell/NATO/


7-11 Oct 1968, Garmisch, Germany
27-31 Oct 1969, Rome, Italy

Download Proceedings, which have transcripts of discussions.
See photographs.

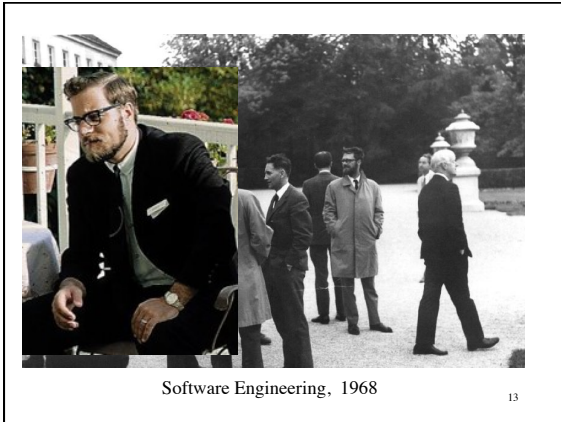
Software crisis:
Academic and industrial people. Admitted for first time that they did not know how to develop software efficiently and effectively.


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Software Engineering, 1968



Next 10-15 years: intense period of research on software engineering, language design, proving programs correct, etc. 12



During 1970s, 1980s, intense research on
 How to prove programs correct,
 How to make it practical,
Methodology for developing algorithms

The way we understand recursive methods is based on that methodology.
 Our understanding of and development of loops is based on that methodology.

Throughout, we try to give you thought habits to help you solve programming problems for effectively

Mark Twain: Nothing needs changing so much as the habits of **others**.

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The way we understand recursive methods is based on that methodology.
 Our understanding of and development of loops is based on that methodology.

Throughout, we try to give you thought habits to help you solve programming problems for effectively

Simplicity is key:
 Learn not only to simplify,
 learn not to complicate.

Separate concerns;
 focus on one at a time.

Develop and test incrementally.

Don't solve a problem until you know what the problem is (give precise and thorough specs).

Learn to read a program at different levels of abstraction.

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