- We can use functions to define *operators* on a set. For example, a *unary operator* on a set A is a function $\sigma : A \longrightarrow A$, (e.g., the 'act of taking negatives' for real numbers, mapping a to $-a$). A *binary* operator is a function $\sigma : A \times A \longrightarrow A$, (e.g., the act of multiplying numbers, so mapping (a, b) to ab). We could have *ternary* operators, or beyond!

- A binary operator is *associative* if $a(bc) = (ab)c \;\; \forall\, a, b, c$. It's *commutative* (also called *Abelian*) if $ab = ba \;\; \forall\, a, b$. It's *idempotent* if $aa = a \;\; \forall\, a$.

- Sometimes it's convenient to notate a function $f : A \times B \longrightarrow C$ as if it were an operator. Let's define the *conjugation* operator $\boldsymbol{\gamma} : A \times B \longrightarrow C$ by $\boldsymbol{\gamma}(a, b) = a^{-1}ba$. For example, suppose we're trying to model the set of all area-preserving transformations of the plane. A little thought shows that these will include all rotations about arbitrary points, all reflections through arbitrary straight lines, and all transpositions (slidings in some direction or other). This could easily get messy! However, adopting some temporary notation, ...

  - Modelling sliding is simply a matter of adding a constant vector amount to every point, so a slide by 6 units horizontally and 5 vertically would be $s : \mathbb{R} \longrightarrow \mathbb{R}$ where $s(x, y) = (x+6, y+5)$. Denote all slides by $\mathcal{S}$.

  - A rotation about an arbitrary point could be represented as a sliding of that point to the origin, then a rotation, then a sliding back. This reduces the issue of modelling all rotations to simply modelling rotations about the origin and using $\mathcal{S}$. Denote all rotations by $\mathscr{T}$ and all rotations about the origin by $\mathcal{R}$, then $\mathscr{T} = \{ s^{-1}rs \mid s \in \mathcal{S},\ r \in \mathcal{R} \} = \boldsymbol{\gamma}(\mathcal{S}, \mathcal{R})$.

  - A reflection through an arbitrary line could be represented as a sliding of that line to pass through the origin (a *nice* line), then a reflection through that nice line, then a sliding back. Denote all reflections (through straight lines) by $\mathcal{F}$, and those whose lines pass through the origin by $\mathcal{N}$. Notice that a general reflection through a straight line passing through the origin can be done by a rotation so the line lays on the x-axis, then an easy reflection (x value stays constant, y value changes sign) followed by undoing the rotation. So if e is the easy reflection, then $\mathcal{F} = \boldsymbol{\gamma}(\mathcal{S}, \mathcal{N}) = \boldsymbol{\gamma}(\mathcal{S}, \boldsymbol{\gamma}(\mathcal{R}, e))$.