CS 2800 Spring 2014
Discrete Mathematics

Topics*:

- Syllogisms and basic logical reasoning
- Set theory
- Algebra (groups, rings, fields and number theory)
- Discrete probability and fuzzy logic
- Formal logic
- Automata
- Graphs

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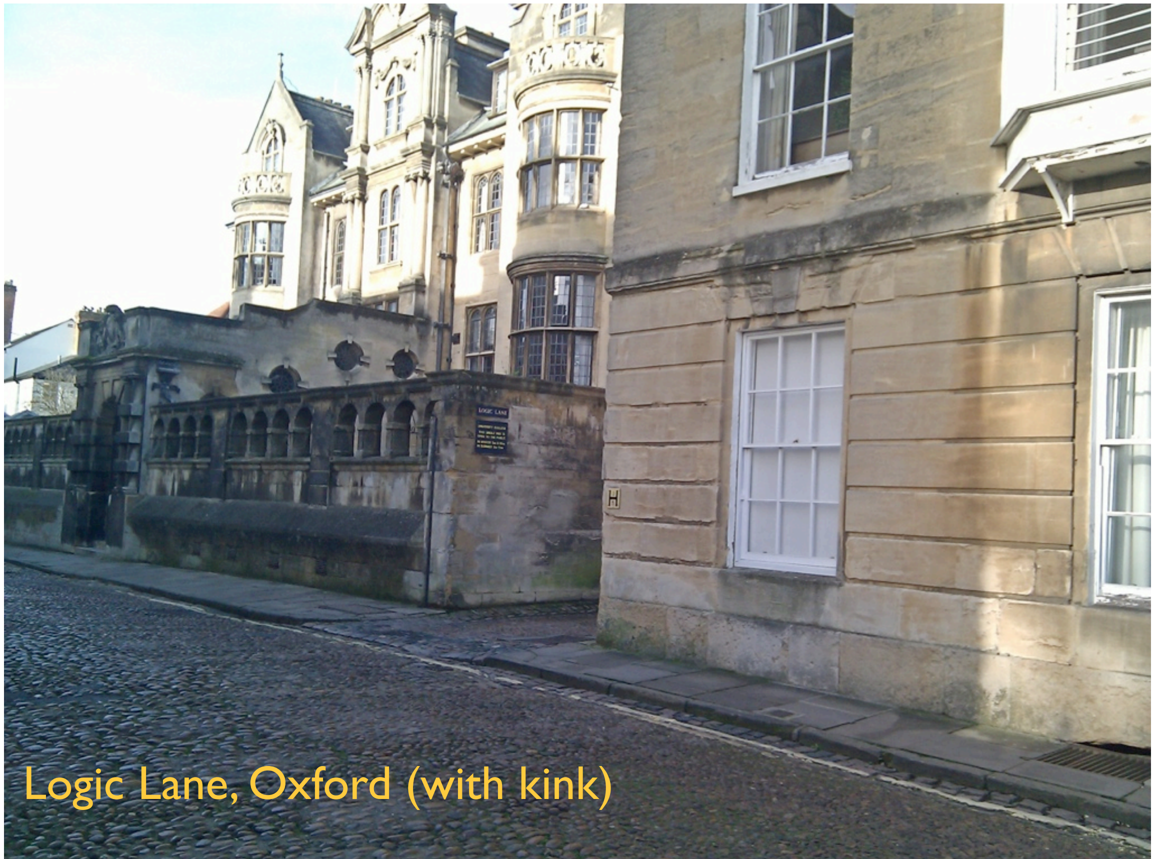
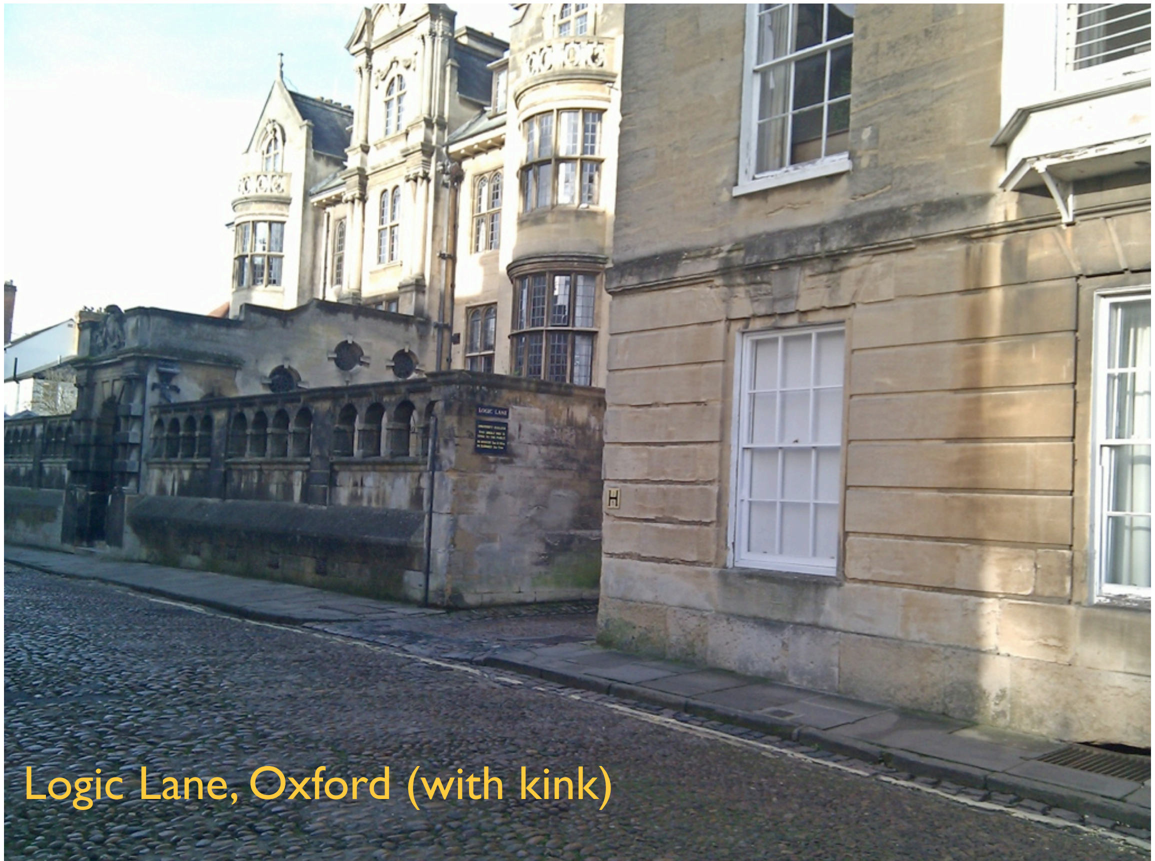
* not necessarily in this order!

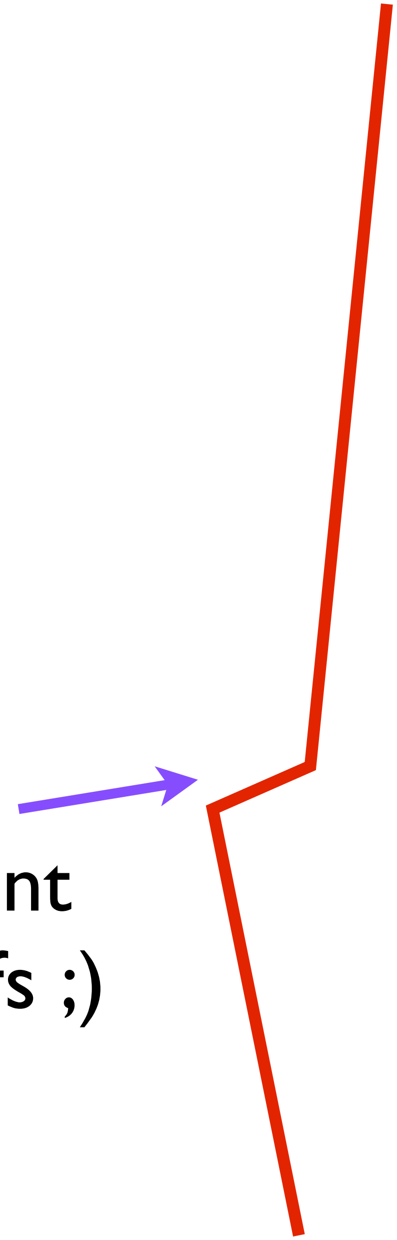


Logic Lane, Oxford (with kink)



Logic Lane, Oxford (with kink)

Most student proofs ;)



- We'll start with some very basic logic, really just refining our reasoning skills.

- Given various boolean statements (A, B, C, etc) we can modify and combine them in various standard ways. We'll often use truth tables to be precise about our definitions, and even to help in proving logical equivalences.

A	~A
T	*F*
F	*T*

  - ***not*** is often notated by ~ (or sometimes !), so if A is true then ~A is false, and vice versa. Notice that ~(~A) = A.
  - ***and*** is often notated by ∧ (or sometimes &), ***or*** is often annotated by ∨ (or sometimes |), ***implies*** is often notated in text by --> (or typeset as ⟹). We use ⟺ to mean *implies and is implied by* (also simply *logically equivalent*, or *if and only if*, which is usually abbreviated to *iff*) which also could be written as (A ⇒ B) ∧ (B ⇒ A).

A	B	A ∧ B
T	*T*	*T*
T	*F*	*F*
F	*T*	*F*
F	*F*	*F*

A	B	A ∨ B
T	*T*	*T*
T	*F*	*T*
F	*T*	*T*
F	*F*	*F*

A	B	A ⇒ B
T	*T*	*T*
T	*F*	*F*
F	*T*	*T*
F	*F*	*T*

A	B	A ⟺ B
T	*T*	*T*
T	*F*	*F*
F	*T*	*F*
F	*F*	*T*

  - The truth table defining *implies* might seem a little odd, but in words this says that if A and B are true then the *implication* from A to B is true, and that if A is true but B is false, then that *implication* cannot possibly be true. The other two choices could be regarded as somewhat arbitrary (albeit regarded as a standard definition!).

- Notice that we can use this approach to demonstrate that certain simplifications work:
  - A ⇒ B is equivalent to ~A ∨ B

A	B	A ⇒ B	~A	~A ∨ B
T	*T*	*T*	*F*	*T*
T	*F*	*F*	*F*	*F*
F	*T*	*T*	*T*	*T*
F	*F*	*T*	*T*	*T*

  - ~(A ∨ B) is equivalent to (~A) ∧ (~B), and ~(A ∧ B) is equivalent to (~A) ∨ (~B)

A	B	A ∨ B	~(A ∨ B)	~A	~B	(~A) ∧ (~B)
T	*T*	*T*	*F*	*F*	*F*	*F*
T	*F*	*T*	*F*	*F*	*T*	*F*
F	*T*	*T*	*F*	*T*	*F*	*F*
F	*F*	*F*	*T*	*T*	*T*	*T*

  - A ∨ (B ∧ C) is equivalent to (A ∨ B) ∧ (A ∨ C). The other distributive rules also hold.

A	B	C	B ∧ C	A ∨ (B ∧ C)	A ∨ B	A ∨ C	(A ∨ B) ∧ (A ∨ C)
T	*T*	*T*	*T*	*T*	*T*	*T*	*T*
T	*T*	*F*	*F*	*T*	*T*	*T*	*T*
T	*F*	*T*	*F*	*T*	*T*	*T*	*T*
T	*F*	*F*	*F*	*T*	*T*	*T*	*T*
F	*T*	*T*	*T*	*T*	*T*	*T*	*T*
F	*T*	*F*	*F*	*F*	*T*	*F*	*F*
F	*F*	*T*	*F*	*F*	*F*	*T*	*F*
F	*F*	*F*	*F*	*F*	*F*	*F*	*F*

- Some further observations before you can have fun with some of Lewis Carroll's syllogisms... We can argue *transitively* in the sense that $A \Rightarrow B$ and $B \Rightarrow C$ yields $A \Rightarrow C$.

- Arguing in the *contrapositive* is really the form of reasoning which understands that the statement $A \Rightarrow B$ is logically equivalent to $\sim B \Rightarrow \sim A$. *

  - Certainly we can prove this via a truth table, but instead let's consider an explicit example. If statement A = "it is raining" and B = "I put up my umbrella", then $A \Rightarrow B$ says that "*IF* it's raining *THEN* I'll put up my umbrella", and $\sim B \Rightarrow \sim A$ says that "*IF* my umbrella isn't up *THEN* it isn't raining". The Latin expression for this is *modus tollens* (aka path of denying).

  - Some people try to claim that $A \Rightarrow B$ is equivalent to $\sim A \Rightarrow \sim B$; this latter expression would be saying that "*IF* it's not raining *THEN* my umbrella won't be up", but that's a false deduction, since I might have my umbrella up as a parasol to protect me from the sun!!! This gets the label *modus morons* (no translation necessary!!).

  - While we're on Latin expressions, the label *modus ponens* (aka path of affirmation) is given to $((A \Rightarrow B) \wedge A) \Rightarrow B$.

- Think about statements such as A = "*every apple is a fruit*", or B = "*there is someone who's stupid*". What are their negations?

  - ~A = "*there is an apple which is not a fruit*", or "*there is at least one apple which is not a fruit*", and ~B = "*there is no-one who's stupid*", or "*every person is not stupid*". Rewriting A as "*for all x which are apples, x is a fruit*" and notating 'every' or 'for all' by $\forall$ (and 'there is' or 'there exists' by $\exists$), we can use a quasi-functional notation to write $A = \forall\, x \in \{\text{apples}\}\ P(x)$ where P(x) = "*x is a fruit*", and so P(x) is some statement involving x.

  - We can summarise these observations by $\sim(\forall\, x \in X\ P(x)) \equiv \exists\, x \in X\ (\sim P(x))$ and $\sim(\exists\, x \in X\ P(x)) \equiv \forall\, x \in X\ (\sim P(x))$.

- We'll say much more about mathematical logic later on in the course, but for now we'll simply observe that there's a theorem that says that essentially logic is the same as set theory, which is the same as number theory. So now it's time to embark on set theory.

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* You might have some fun looking at these expressions by treating the operators $\vee$ like plus and $\wedge$ like multiplication, and applying this to the numbers 0 and 1, defining addition to yield the remainder of the sum after dividing by 2 (i.e., addition mod 2).