Relational Graph Models, Taylor Expansion and Extensionality

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Université Paris-Nord – Paris 13 (France)

MFPS XXX
Ithaca, New York
15th June 2014

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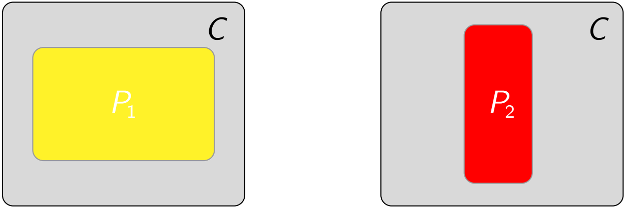

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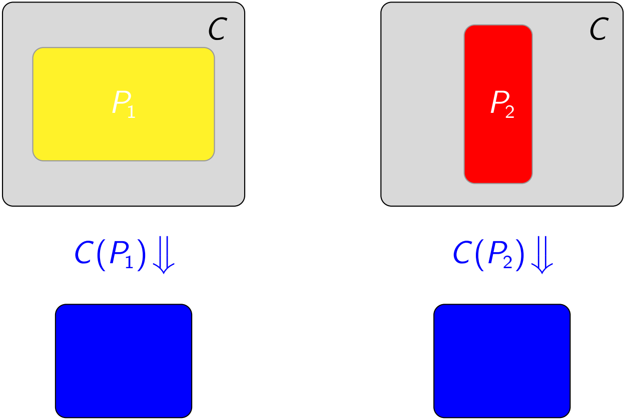

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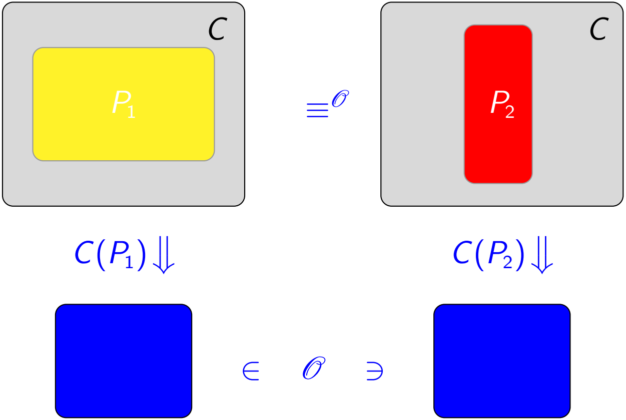

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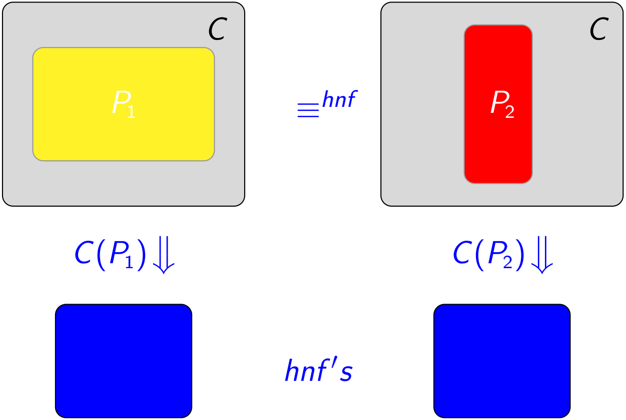


Morris's observational equivalence for the untyped λ-calculus

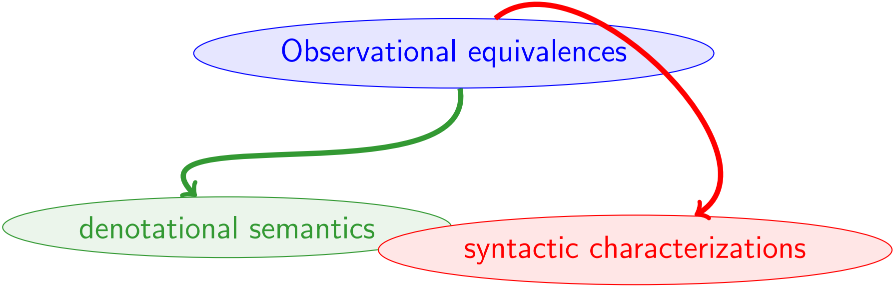
$$M \equiv^{\mathrm{nf}} N$$

$$\Updownarrow$$

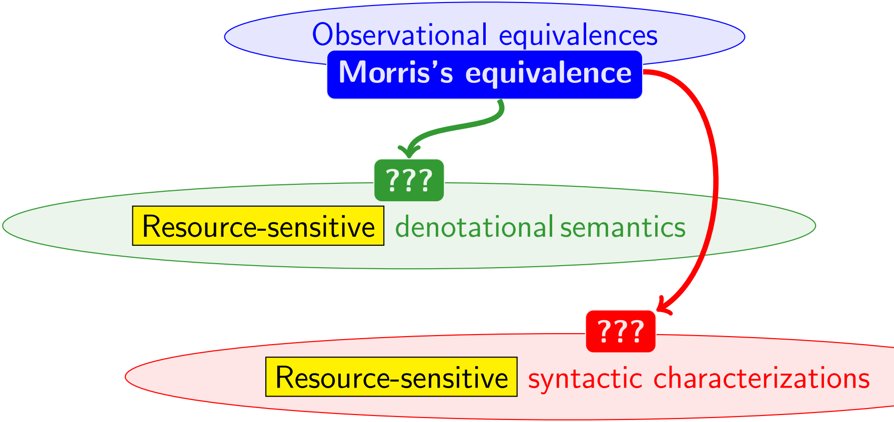
for all context $C(-)$

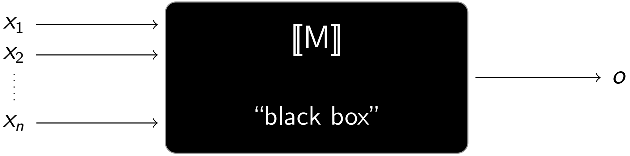
$C(M)$ has a β-normal form iff $C(N)$ has a β-normal form

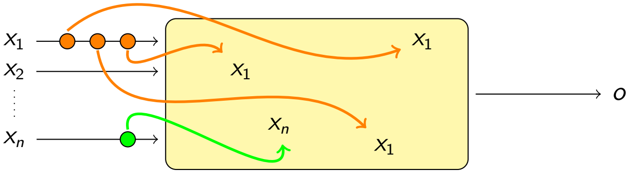
Characterizing observational equivalences



Resource-sensitive characterizations for Morris

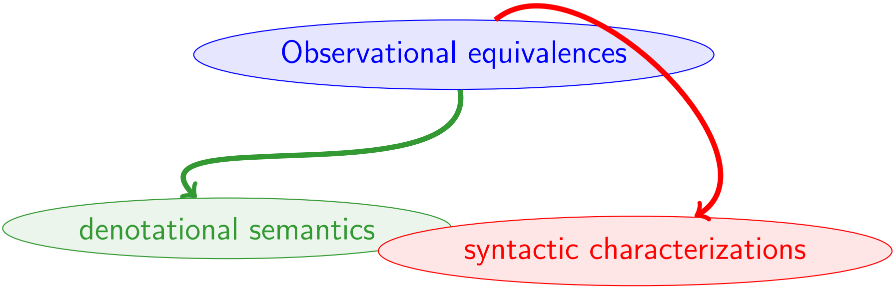



x_1
x_2
x_n
⟦M⟧
“black box”
o



What about resource usage of the execution ?

Characterizing observational equivalences



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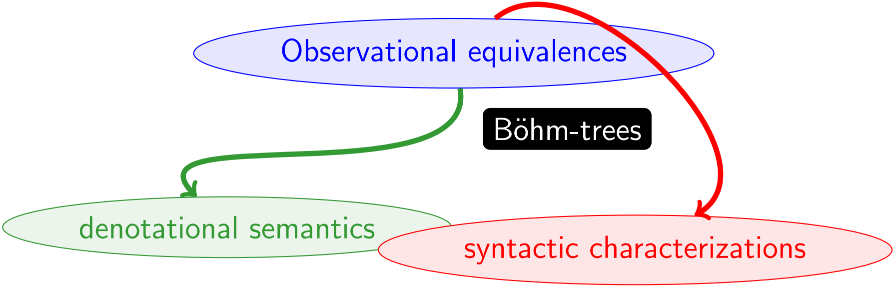
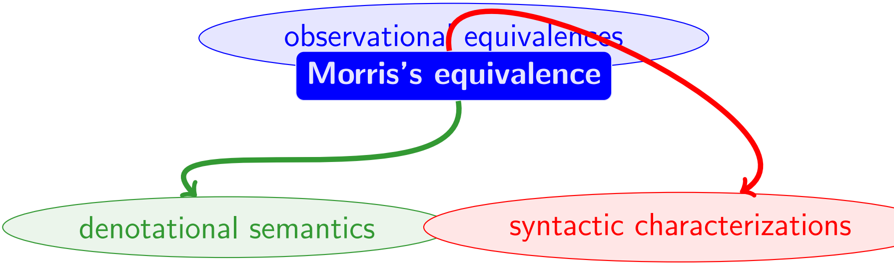


[Barendregt 1981]

Characterizing Morris's equivalence



[Morris 1968, Hyland 1975]

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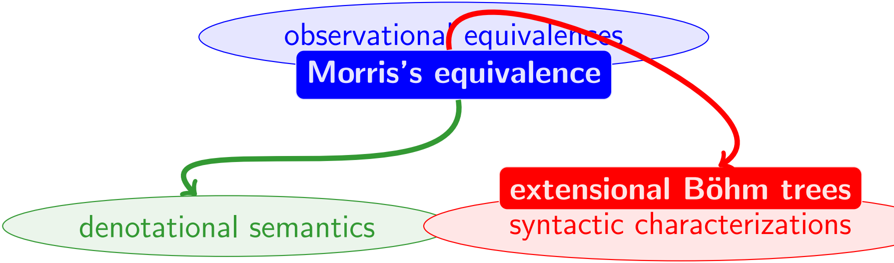

[Hyland 1975, Levy 1976]

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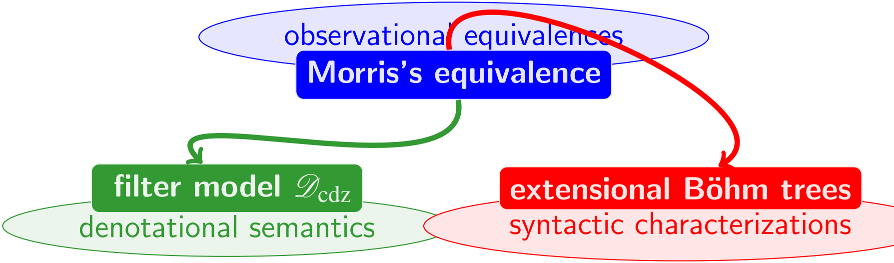

[Coppo, Dezani & Zacchi 1987]

The key idea

Böhm tree

Output of the program

Approximations of the β-normal form of the term

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Taylor expansion

Resource management of the program

$$\mathscr{T}(MN) = \sum_{k \in \mathbb{N}} M[\overbrace{N, \ldots, N}^{k \text{ times}}]$$

[Ehrhard & Regnier 2003]

(Linear) Resource Calculus

$$\begin{array}{rcll} t & ::= & x \mid \lambda x.t \mid t\,b & \\ b & ::= & [t_1, \ldots, t_n] & \text{where } n \geq 0 \end{array}$$

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Terms

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Bags

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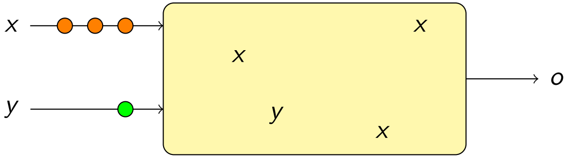
$$(\lambda x \lambda y.t)[s_{11}, s_{12}, s_{13}][s_{21}]$$



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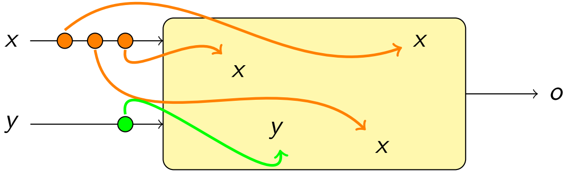

(Linear) Resource Calculus

If the number of occurrences of x in t equals k

$$(\lambda x.t)[s_1, \ldots, s_k] \quad \to_\beta \quad \sum_{p \in \mathfrak{S}_k} t\left\{s_{p(1)}/x_1, \ldots, s_{p(k)}/x_k\right\}$$

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Seeking linear substitution?

Then take non-determinism, too!

Taylor Expansion

$\mathscr{T} \quad : \quad \lambda\text{-terms} \quad \rightarrow \quad$ infinite sums of resource terms

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Taylor Expansion

$\mathscr{T} \quad : \quad \lambda\text{-terms} \quad \to \quad \text{infinite sums of resource terms}$

$$\mathscr{T}(MN) = \sum_{k \in \mathbb{N}} M[\overbrace{N, \ldots, N}^{k \text{ times}}]$$ I was lying!

Definition (Taylor expansion)

$$\mathscr{T}(x) = x \qquad \mathscr{T}(\lambda x.M) = \sum_{t \in \mathscr{T}(M)} \lambda x.t$$

$$\mathscr{T}(MN) = \sum_{k \in \mathbb{N}, t \in \mathscr{T}(M), s_1, \ldots, s_k \in \mathscr{T}(N)} t[s_1, \ldots, s_k]$$

Taylor Expansion

$\mathscr{T} \;:\; \lambda\text{-terms} \;\rightarrow\; \text{infinite sums of resource terms}$

Example

$$\mathscr{T}(\lambda y.xyy) = \sum_{n,k\in\mathbb{N}} \lambda y.x\,[\overbrace{y,\ldots,y}^{n \text{ times}}]\,[\overbrace{y,\ldots,y}^{k \text{ times}}]$$

Taylor Expansion

$\mathscr{T}$: λ-terms $\rightarrow$ infinite sums of resource terms

Theorem (Ehrhard & Regnier 2008)

For every λ-term M

$$\mathrm{nf}_\beta\big(\mathscr{T}(M)\big) = \mathscr{T}(\mathrm{BT}(M)).$$

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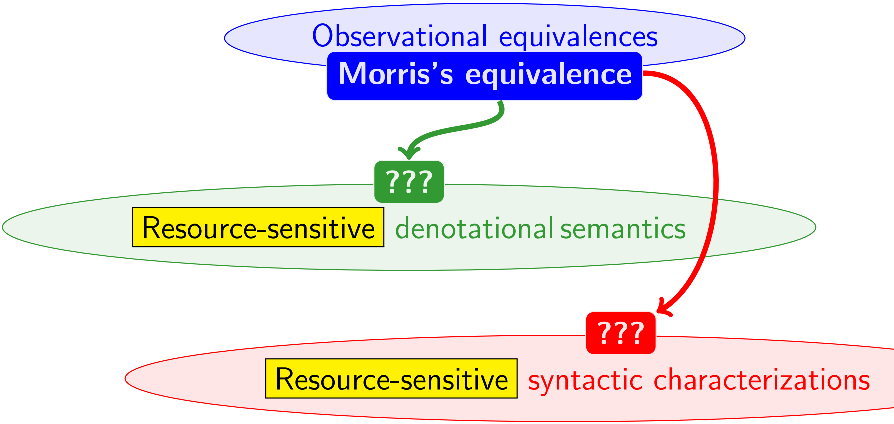

Relational Semantics

The cartesian closed category **MRel**:

objects:	sets $A, B, \ldots$
morphisms:	$A \to B \;\subseteq\; \mathscr{M}_{\mathrm{f}}(A) \times B$

Kleisli of **Rel**, self-dual $\star$-autonomous category, provided with the exponential comonad $! = \mathscr{M}_f$.

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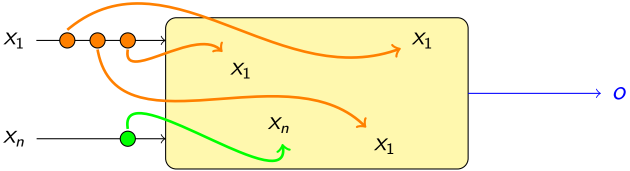

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$$[D \Rightarrow D] \quad \lhd \quad D$$

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Relational Graph Models

Definition (relational graph model)

$\mathscr{D} = (D, i)$ is given by

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- a total injection $i : \mathscr{M}_{\mathrm{f}}(D) \times D \to D$.

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Reminder (Plotkin's graph model)

$\mathscr{D} = (D, i)$ is given by

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rgm as *Non Idempotent* Intersection Type Systems

To a rgm $\mathscr{D} = (D\,,\, i)$ we associate

$\mathsf{T}_{\mathscr{D}}: \quad \sigma \ ::= \ \alpha \ | \ \mu \to \sigma \qquad\qquad \mathsf{I}_{\mathscr{D}}: \quad \mu \ ::= \ \omega \ | \ \sigma \ | \ \sigma \wedge \mu$

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The injection $i : \mathscr{M}_{\mathrm{f}}(D) \times D \to D$ induces an equivalence

$$\sigma \simeq \tau$$

rgm as *Non Idempotent* Intersection Type Systems

For a closed λ-term M

$$[\![M]\!]^{\mathscr{D}} = \{\sigma \mid \ \vdash^{\mathscr{D}} M : \sigma\}$$

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Theorem (approximation)

$$[\![M]\!]^{\mathscr{D}} = \left[\!\!\left[\, \mathrm{BT}(M) \,\right]\!\!\right]^{\mathscr{D}}$$

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Theorem (New approximation)

$$[\![M]\!]^{\mathscr{D}} = \left[\!\!\left[\ \mathscr{T}(M)\ \right]\!\!\right]^{\mathscr{D}} = \left[\!\!\left[\ \mathrm{BT}(M)\ \right]\!\!\right]^{\mathscr{D}}$$

It is *here* that we replace the traditional Böhm tree -like approximations with the new Taylor -like one!

Full Abstraction

Theorem (that we are looking for)

Let $\mathscr{D}$ be a rgm with possibly some more hypothesis. The following are equivalent for M, N closed λ-terms:

1. $[\![M]\!]^{\mathscr{D}} = [\![N]\!]^{\mathscr{D}}$
2. $M \equiv^{\mathrm{nf}} N$

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Lemma (characterization of β-normalizability)

A closed λ-term M has a β-normal form iff in every rgm $\mathscr{D}$ preserving ω-polarities,

$$\vdash^{\mathscr{D}} M : \sigma$$

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Examples

[Bucciarelli & others 2007]

$\mathscr{D}_\infty$	ε	$\omega \to \varepsilon \simeq \varepsilon$	fully abstract for $=^{hnf}$
$\mathscr{D}_\star$	$\star$	$\star \to \star \simeq \star$	fully abstract for Morris!

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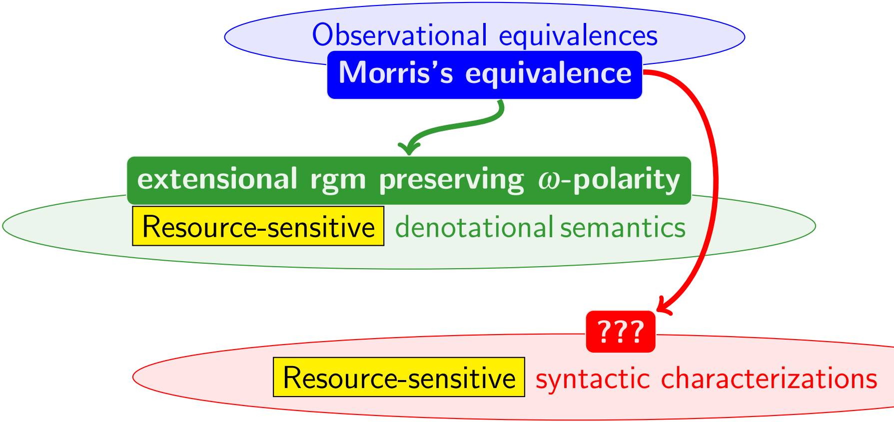

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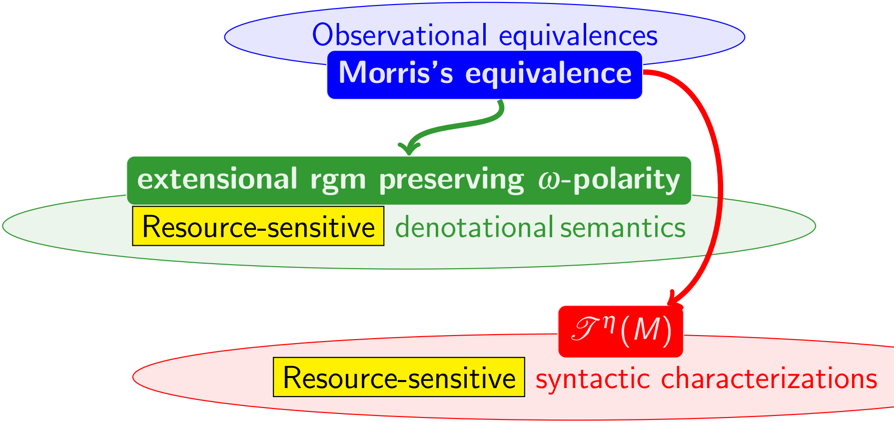

Extensional Taylor Expansion

We seek an analogous of [Ehrhard & Regnier 2008] "commutativity" theorem

$$\mathrm{nf}_\beta\left(\mathscr{T}(M)\right) = \mathscr{T}\left(\mathrm{BT}(M)\right)$$

taking η-reduction into account. Something like

$$\mathrm{nf}_\eta\left(\mathrm{nf}_\beta\left(\mathscr{T}(M)\right)\right) = \mathscr{T}\left(\mathrm{BT}^e(M)\right)$$

Extensional Taylor Expansion

The core problem: how to η-reduce resource terms?
A paradigmatic instance of the problem:

$$\sum_{n\in\mathbb{N}} \lambda x.y\,[\overbrace{x,\ldots,x}^{n\ \text{times}}] \quad \to_\eta \quad y \qquad ?$$

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$$\mathscr{T}^{\eta}(M) = \mathrm{nf}_{\eta^{\ell}}\mathscr{L}\left(\mathrm{nf}_{\beta}\, \mathscr{T}(M)\right)$$

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$$\mathscr{T}^{\eta}(M) = \mathrm{nf}_{\eta^{\ell}}\mathscr{L}\left(\mathrm{nf}_{\beta}\,\mathscr{T}(M)\right)$$

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Extensional Taylor Expansion

Theorem

For every λ-term M

$$\mathscr{T}^{\eta}(M) = \mathscr{T}\,\mathrm{BT}^{\eta}(M)$$

Here $\mathrm{BT}^{\eta}(M)$ is a coinductive version of the extensional Böhm tree, that characterizes $\equiv^{\mathrm{nf}}$, but *not* $\sqsubseteq^{\mathrm{nf}}$.

So do our $\mathscr{T}^{\eta}(M)$. Better than nothing!

Conclusion

- We introduced a class of models fully abstract for $\sqsubseteq^{\mathrm{nf}}$

  extensional rgm preserving ω-polarity

- We introduced a syntactic mathematical model of $\equiv^{\mathrm{nf}}$

  $\mathscr{T}^{\eta}(M)$

Conclusion & Future Work

- We introduced a class of models fully abstract for $\sqsubseteq^{\mathrm{nf}}$

  extensional rgm preserving ω-polarity

- Future works: refining the notion of preservation of ω-polarity in order to get *all* relational models fully abstract for $\sqsubseteq^{\mathrm{nf}}$, along the line of [Breuvart LICS14]

- We introduced a syntactic mathematical model of $\equiv^{\mathrm{nf}}$

  $\mathscr{T}^{\eta}(M)$

- Future works: looking for a similar model of $\sqsubseteq^{\mathrm{nf}}$

Thanks for your attention.