Modeling the Reliability of Packet Group Transmission in Wireless Network

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***Abstract*— Most previous models about wireless channel are proposed to capture long-time channel characteristics. However, when we combine group transmission with error correcting mechanisms, the steady-state probabilities are no longer accurate enough to depict the short-time loss states. In this paper, we propose a new analytical model for group transmission which can capture influences of initial channel state and group length on transmission reliability. The model offers us a significant insight into loss characteristics of group transmission, which is essential to design reliable wireless protocols. Finally to illuminate the strength of our model, we also apply our model to compare the reliability performance of multipath transmission with single path transmission.**

***Index Terms*—Reliability, packet group transmission, Gilbert model, correlated wireless channel**

I. INTRODUCTION

Driven by industrial and scientific applications, packet-loss or bit-error modeling of wireless network has recently attracted much attention from academia.

For unpredictable wireless channels, a lot of models are proposed [1]-[4], which offer us a significant insight into characteristics of the underlying wireless channels. In most previous models, a completely accurate analysis of the error process could be complicated and often only the long-time statistics can be computed. For instance, Markov models are generally employed to characterize error processes. The steady-state probabilities of a Markov model, which represent the long-run proportion of the time spent in each state, are key parameters in modeling.

However, when we adopt packet group transmission based on Erasure coding [5], [6] or FEC [7], the *homogeneousness* among packets is modified and steady-state probabilities are not accurate enough to depict the loss event. For example, in a channel with steady loss probability 0.2, isolated loss and burst loss make no difference for the reliability of mass transmission (e.g. more than thousands of packets) while having a great impact on the reliability of group transmission. As shown in Fig. 1, based on the loss probability, the corresponding Erasure coding [M=4: R=1] can recover all isolated losses but only some burst losses.

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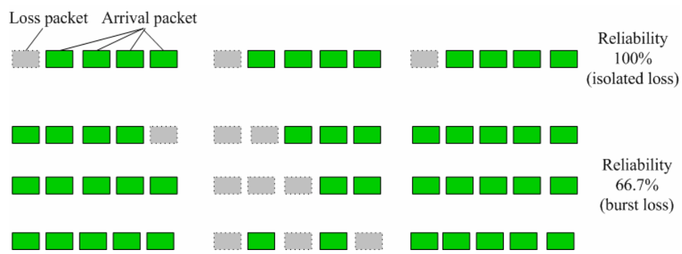

Fig. 1. Group Transmission using Erasure Coding

Hence in unreliable wireless network, the correlation between packets in one group has to be considered and the short-time statistic is indispensable to design an efficient and reliable mechanism. Especially in low-rate Wireless Sensor Networks, the data traffic generated by one sensor may be of very low intensity but burst traffic may be triggered by a set of sensors due to a common event. Thus the design based on the long-time statistics is no longer accurate and our work is motivated by the need to understand the short-time loss behavior in wireless network.

In this paper we propose a new model, based on the Gilbert theory [8], to compute of the loss probability of packet group. Our contribution can be summarized as follows:

1) Extend the Gilbert model to capture the influence of initial channel states on transmission reliability. Since most previous models are utilized for long-time statistics, they paid little attention to the influence of initial channel state. However, when considering the short-length group transmission, we can no longer ignore that point. Their importance is demonstrated with theoretical analysis.
2) With the help of the new model, we provide a theoretical study about how to define reliable protocols based on packet group transmission. In traditional one-by-one transmission, large numbers of packets can ensure the accuracy of long-time statistic. However, in low-rate wireless networks (e.g. WSNs) that require high reliability, correlations inside group are not negligible, which makes long-time statistics lose accuracy. Therefore in this paper, we deliberately explore the relationship between reliability and group length.
3) To illuminate the value of our model, we also apply our model to multipath scheme and discover that multipath is not always more reliable than single path transmission.

The rest of this paper is outlined as follows. In Section II we briefly introduce the basic wireless channel model, which is a prerequisite theoretical foundation for the whole paper. Then we provide a new analytical model to represent reliability of packets in groups under different parameters in Section III. The theoretical evaluation is carried out in Section IV. In Section V we compare multipath scheme with single path scheme based on our new model. Conclusions are given in Section VI.

II. Preliminaries

Errors or losses occur on the wireless channel due to various impairments including interference and mobility, which exhibit some degree of correlations. Markov models have been widely used to characterize loss behavior [1], [2]. Considering the tractability and accuracy in low-rate network of the Gilbert Model [9], it is adopted in this paper as a basic model.

In the Gilbert Model (Fig. 2), p is the transition probability of going from a non-loss state to a loss state and q is the probability of going from a loss sate to a non-loss state. 1-q, also called conditional loss probability (*clp*) is the probability that the next packet is again lost, provided the previous one has been lost.

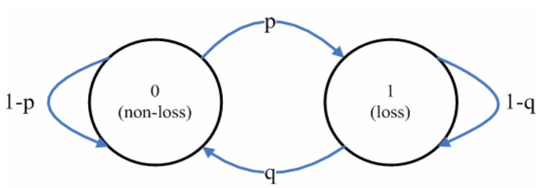

Fig. 2. The Gilbert model

The stationary probabilities of the Gilbert model represent the long-time proportion of the time spent in each state. Once the transitional probabilities are known, we can compute stationary probability π_0 for non-loss state and π_1 for loss state, which is also called the unconditional loss probability (*ulp*):

$$\pi_0 = \frac{q}{p+q}, \pi_1 = \frac{p}{p+q}. \tag{1}$$

In other words, π_0 and π_1 also represents the mean arrival and loss probability. From collected network traffic traces, we can easily obtain trained parameters p and q.

III. Theoretical Model

Since traditional methods (e.g. Gilbert model) use long-time stationary probabilities to represent packet-loss or bit-error processes, it is hard to characterize the dynamic factors in heterogeneous group transmission. In this section we propose a new analytical model, which can capture the influence of initial channel state and group length on transmission reliability.

For N packets transmitted in one group, each packet can transmit successfully or suffer loss event. We define $\{s(t), t=1..N\}$ as the stochastic process to represent transmission state of N packets, then packets series model is constructed in Fig. 3. Here the transitional probabilities p and q are both the same as the Gilbert model, representing one-step state transition probability between loss state and non-loss state.

We define probabilities for loss and non-loss state of i-th pa-

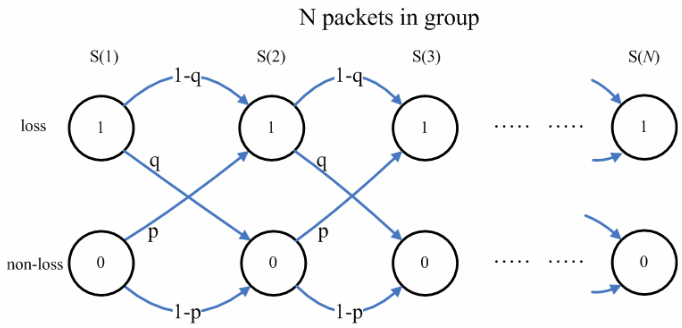

Fig. 3. Packets series model

-cket as:

$$\alpha_i = P\{s(i)=1\}, \beta_i = P\{s(i)=0\}, (i \in [1,N]) \tag{2}$$

Specifically, we set the initial probabilities for the first packet as

$$\alpha_1 = a, \beta_1 = 1-a \tag{3}$$

From the model in Fig. 3, it is observed that

$$\begin{pmatrix}\alpha_k\\ \beta_k\end{pmatrix} = \begin{pmatrix}1-q & p\\ q & 1-p\end{pmatrix}\begin{pmatrix}\alpha_{k-1}\\ \beta_{k-1}\end{pmatrix} \tag{4}$$

Then we can get

$$\begin{pmatrix}\alpha_k\\ \beta_k\end{pmatrix} = A^{k-1} * \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix}, (k \geq 1, A^0 = I) \tag{5}$$

where $A = \begin{pmatrix}1-q & p\\ q & 1-p\end{pmatrix}$.

Let E_l be the mean loss number and E_n be the mean arrival number, which can be calculated as

$$\begin{pmatrix}E_l\\ E_n\end{pmatrix} = \sum_{i=1}^{N}\begin{pmatrix}\alpha_i\\ \beta_i\end{pmatrix} = \left(\sum_{i=1}^{N} A^{i-1}\right) * \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \tag{6}$$

To simplify the above formula, we utilize Jordan Normal Form of A as

$$A = S * J * S^{-1} \tag{7}$$

where

$$J = \begin{pmatrix}1 & 0\\ 0 & 1-p-q\end{pmatrix}$$

and

$$S = \begin{pmatrix}\frac{p}{p+q} & \frac{q}{p+q}\\ \frac{q}{p+q} & \frac{-q}{p+q}\end{pmatrix}.$$

Based on the Jordan Normal Form, we can rewrite (6) as

$$\begin{pmatrix}E_l\\ E_n\end{pmatrix} = S * \left(\sum_{i=1}^{N} J^{i-1}\right) * S^{-1} * \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \tag{8}$$

Owing to the fact that

$$\sum_{i=1}^{N} J^{i-1} = \begin{pmatrix}N & 0\\ 0 & \frac{1-(1-p-q)^N}{p+q}\end{pmatrix}, \tag{9}$$

we have

$$\begin{pmatrix}E_l\\ E_n\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} * \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \tag{10}$$

where

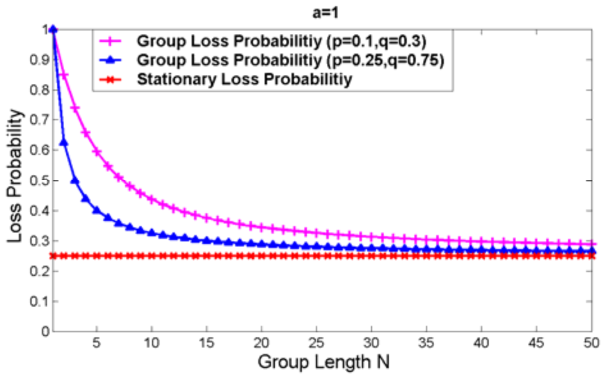

Fig. 4. Initial Channel State a=1

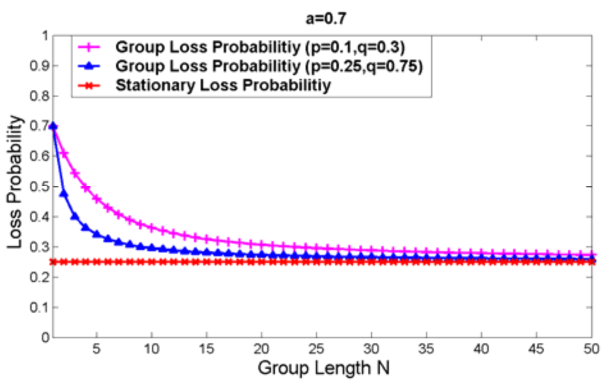

Fig. 5. Initial Channel State a=0.7

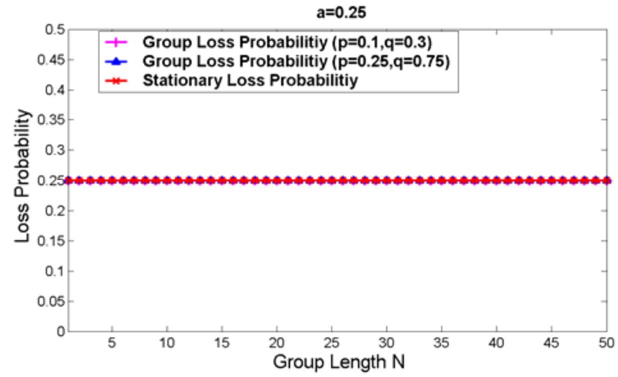

Fig. 6. Initial Channel State a=0.25

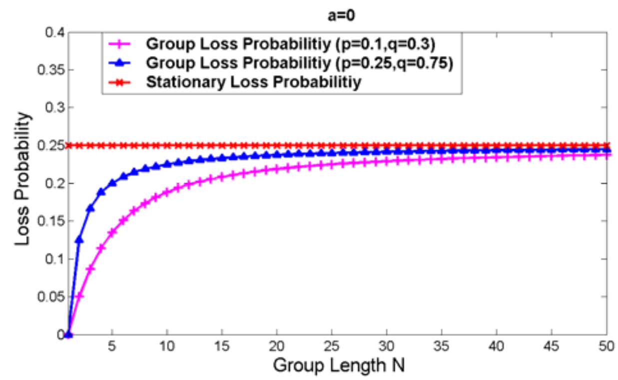

Fig. 7. Initial Channel State a=0

$$\begin{cases} a_{11} = \dfrac{q-(1-p-q)^N q + Np(p+q)}{(p+q)^2} \\ a_{12} = \dfrac{p(-1+(1-p-q)^N + N(p+q))}{(p+q)^2} \\ a_{21} = \dfrac{q(-1+(1-p-q)^N + N(p+q))}{(p+q)^2} \\ a_{22} = \dfrac{p-(1-p-q)^N p + Nq(p+q)}{(p+q)^2} \end{cases}$$

Then the mean number of loss packets E_l can be calculated as

$$\begin{aligned} E_l &= \alpha_1 a_{11} + \beta_1 a_{12} \\ &= \frac{(-1+(1-p-q)^N)\times((1-a)p-aq)+Np(p+q)}{(p+q)^2}. \end{aligned} \tag{11}$$

And the mean loss probability of one group can be expressed as

$$\eta = \frac{(-1+(1-p-q)^N)\times((1-a)p-aq)+Np(p+q)}{N(p+q)^2}. \tag{12}$$

Obviously from (12), we can figure out the mean loss probability in packet group transmission is determined by transitional probabilities (i.e. p and q), the group length N and the initial probabilities a.

And as the group length N approaches infinity, the limit of η is

$$\lim_{N\to\infty} \eta = \frac{Np(p+q)}{N(p+q)^2} = \frac{p}{(p+q)} = \pi_1, \tag{13}$$

which is exactly the mean loss probability π_l in (1). This result can denote that our model is fine-granularity application of the Gilbert model.

IV. Numerical Results

In this section, the model proposed above is utilized to analyze the reliability of packets group transmission under different conditions. Due to length constraint, here we only discuss the case of stationary packet loss probability π_l=0.25 without loss of generality. Under this stationary probability, the loss results under different initial channel state (a=1, 0.7, 0.25 and 0) are shown from Fig. 4 to Fig. 7.

Fig. 4 shows loss probabilities of group transmission for two channel states (namely, different transitional probabilities p and q) under the same initial channel state. It is observed that in such a bad initial channel condition (a=1), the reliability of group transmission is extraordinary unacceptable when group length N is smaller than 5. However with the increase of N, the loss probabilities of group transmission gradually coincide with the stationary loss probability 0.25.

Additionally, we also note from Fig. 4 that the smaller the transitional probabilities are, the slower the change of loss probabilities are. For example, the loss result of p=0.1 and q=0.3 alters more slowly than that of p=0.25 and q=0.75. This conclusion results from that small transitional probabilities, which means high correlation between packets, will enhance the channel memory.

Similar with Fig. 4, the results from Fig. 5 also represent the influence of group length N under initial channel state. Compared with results when a=1, a better initial channel state a=0.7 will result in better reliability with the same group length.

In Fig. 6, it is obviously that when initial channel condition a is equal to stationary packet loss probability π_1, the result of our model is exactly the same with the long-time statistic value. And no matter how long the group length is, the reliability of group transmission is always stable.

To further investigate the influence of the initial channel condition on group reliability, we report results in Fig. 7 under an ideal initial channel state (a=0). Due to the channel memory, the reliabilities of group transmission behave better than steady state, especially when the group length N is small. Furthermore we can also notice that the smaller transitional probabilities are, the more apparent the channel memory is.

From above analysis, it is clearly observed that our new model does reflect the influence of the initial channel state and the group length. Our theoretical results verify that in order to obtain high reliability of group transmission in bursty or unstable wireless network, we need increase the group length or add redundant packet to enhance resistance against loss events. And the design based on long-time statistics cannot guarantee reliability in complicated and unpredictable wireless network.

V. MULTIPATH STUDY

To illuminate the value of our model, we apply our model to multipath transmission in this section. The application of multipath technique in wireless network seems nature, since it may diminish the effect of unstable wireless links to increase reliability. Thus most previous work adopt mechanism which combine path redundancy (i.e. multiple disjoint paths) and data redundancy (e.g. Erasure codes or FEC) to provide reliable transmission [7], [10]-[12]. However based on our model, we can discover some facts that are neglected before.

In our reliability comparison we assume t paths are available for tN packets transmitted from a source to a destination node. Each path has the same transitional probabilities, which result in the same long-time steady state, and different initial channel state a_i. Without loss of generality, it is assumed that

$$a_1 \ge a_2 \ge \dots \ge a_t \, . \tag{14}$$

Additionally, load balancing is implemented in multipath transmission to ensure N packets are allocated evenly in each single path. Using our model, we can get the following theorem:

Theorem 1: The multipath scheme equals or is better than the worst single-path one when transmitting large numbers of packets.

Proof:

Based on (12), the loss probabilities of tN packets in single-path transmission can be represented as

$$\eta_i(tN) = \frac{(1-(1-p-q)^{tN}) \times (a_i(p+q)-p) + tNp(p+q)}{tN(p+q)^2} \, . \tag{15}$$

Owing to the linear relationship between a_i and η_i, we can get

$$\eta_1(tN) \ge \eta_2(tN) \ge \dots \ge \eta_t(tN) \, . \tag{16}$$

Using multipath transmission with load balancing, tN packets are allocated evenly in t single-path, then the loss probabilities of tN packets in multipath transmission is

$$\eta_{mul} = \frac{\sum_{i=1}^{t} N \times \eta_i(N)}{tN} = \frac{\sum_{i=1}^{t} \eta_i(N)}{t} \ge \frac{t \times \eta_t(N)}{t} = \eta_t(N) \, . \tag{17}$$

When the group length N approaches infinity, there exist

$$\lim_{N\to\infty} \eta_t(tN) = \lim_{N\to\infty} \eta_t(N) \, . \tag{18}$$

Refer to (17) and (18), we finally obtain

$$\eta_{mul} \ge \eta_t(tN) \, . \tag{19}$$

This theorem does explain why it is generally agreed that multipath transmission can diminish the effect of unstable wireless links to increase reliability.

However, when we set initial channel state a_i as the same fixed value a for all multiple paths, we discover another fact.

Theorem 2: The multipath scheme is not always more reliable than the single-path transmission.

Proof:

Since $a = a_1 = a_2 = \dots = a_t$, the loss probabilities of tN packets in single-path transmission can be represented as

$$\eta_{sig} = \eta(tN) \, , \tag{20}$$

and the loss probabilities in multipath transmission is

$$\eta_{mul} = \frac{\sum_{i=1}^{t} N \times \eta_i(N)}{tN} = \frac{\sum_{i=1}^{t} \eta_i(N)}{t} = \eta(N) \, . \tag{21}$$

Then difference of reliability is

$$\begin{aligned}\Delta\eta &= \eta_{sig} - \eta_{mul} \\ &= \frac{(p-a(p+q))((1-p-q)^{tN} - 1 + t - t(1-p-q)^{N})}{tN(p+q)^2} \, .\end{aligned} \tag{22}$$

For convenience of discussion, we set t=3 and rewrite (22) as

$$\begin{aligned}\Delta\eta &= \eta_{sig} - \eta_{mul} \\ &= \frac{(p-a(p+q))((1-p-q)^{3N} - 3(1-p-q)^{N} + 2)}{3N(p+q)^2} \, .\end{aligned} \tag{23}$$

It is not difficult to prove that when $p, q \in [0,1]$, we have

$$(1-p-q)^{3N} - 3(1-p-q)^{N} + 2 \ge 0 \, . \tag{24}$$

Here the value of $p - a(p+q)$ is the key determinant to the final result. Specifically when $p \ge a(p+q)$, $\Delta\eta \ge 0$ which means the multipath transmission is more reliable; Or else single-path scheme provides more reliable packet transmission.

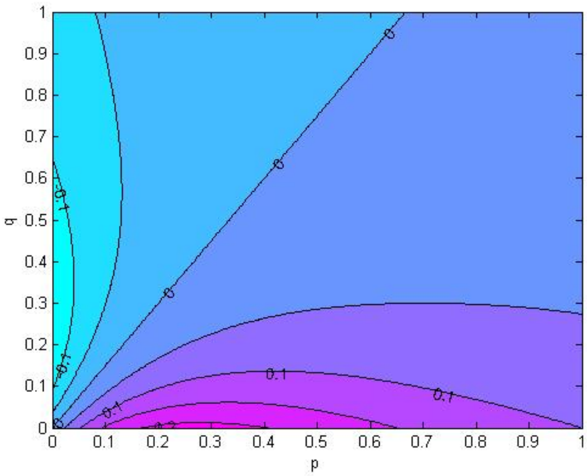

Fig. 8. a=0.4, N=4

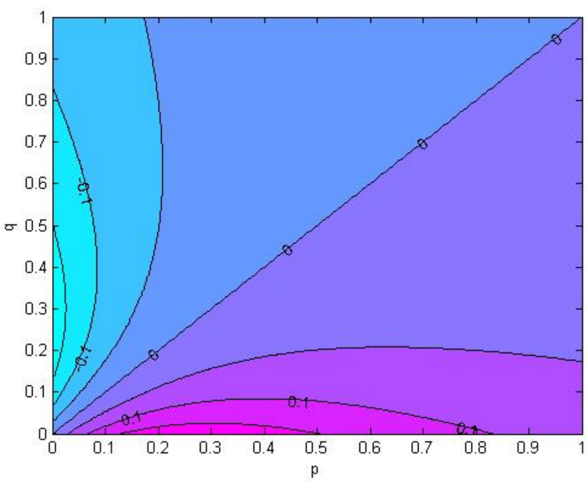

Fig. 9. a=0.5, N=4

From Fig. 8 and Fig. 9, it is clearly observed the influence of initial channel state on result. We call the area above contour line "0" as *single-path superior region* and the area below as *multipath superior region.* From the movement of contour line "0" in two figures, we can find that worse initial channel state (a=0.5) will cause smaller *multipath superior region,* which is to verify that due to the effect of correlation, the multipath scheme is not always more reliable than the single-path transmission.

Finally as N approaches infinity, the limit of $\Delta\eta$ is

$$\lim_{N\to\infty} \Delta\eta = \lim_{N\to\infty} \eta(tN) - \lim_{N\to\infty} \eta(N) = 0\,. \tag{25}$$

When comparing Fig. 8 and Fig. 10, we can figure out increase of N will reduce the reliability difference of two schemes.

VI. Conclusion

In this paper, we propose a simple but tractable model to analyze the loss probability for packet group transmission. Our results show that in bad initial channel states, high correlation among packets in group will largely deteriorate the reliability performance. However, with the increase of the group length, the impact of correlation gradually diminishes. Finally we apply our model to multipath transmission and reveal that ow-

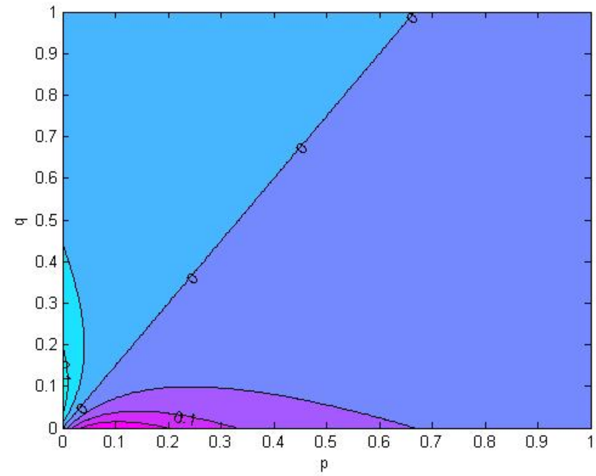

Fig. 10. a=0.4, N=12

-ing to the correlation effect, namely channel memory, the multipath scheme behaves even worse that single-path under bad initial channel state.

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