

TABLE OF CONTENTS

List of Algorithms	iv
List of Figures	v
List of Tables	vi
Chapter 1: Introduction	1
1.1 Theoretical Contributions	3
1.1.1 The Resolution Complexity of Structured Problems	4
1.1.2 Hardness of Approximation	4
1.2 Proof Systems Underlying SAT Solvers	5
1.2.1 Clause Learning, Restarts, and Resolution	5
1.3 Building Faster SAT Solvers	6
1.3.1 Variable Ordering Using Domain Knowledge	6
1.3.2 Utilizing Structural Symmetry in Domains	7
Chapter 2: Preliminaries	8
2.1 The Propositional Satisfiability Problem	9
2.2 Proof Systems	9
2.2.1 Resolution	10
2.2.2 Refinements of Resolution	11
2.2.3 The Size-Width Relationship	12
2.3 The DPLL Procedure and Clause Learning	13
2.3.1 Relation to Tree-like Resolution	15
2.3.2 Clause Learning	16
Chapter 3: The Resolution Complexity of Graph Problems	18
3.1 Independent Sets in Random Graphs	21
3.2 Encoding Independent Sets as Formulas	23

3.2.1	Encoding Based on Counting	25
3.2.2	Encoding Based on Mapping	26
3.2.3	Encoding Using Block-respecting Independent Sets	27
3.2.4	Relationships Among Encodings	29
3.3	Simulating Chvátal’s Proof System	31
3.4	Relation to Vertex Cover and Coloring	34
3.4.1	Vertex Cover	34
3.4.2	Coloring	37
3.5	Upper Bounds	38
3.6	Key Concepts for Lower Bounds	42
3.7	Proof Sizes and Graph Expansion	44
3.7.1	Relating Proof Size to Graph Expansion	44
3.7.2	Lower Bounding Sub-critical Expansion	46
3.8	Lower Bounds for Resolution and Associated Algorithms	50
3.9	Hardness of Approximation	52
3.9.1	Maximum Independent Set Approximation	53
3.9.2	Minimum Vertex Cover Approximation	54
3.10	Stronger Lower Bounds for Exhaustive Backtracking Algorithms and DPLL	57
3.11	Discussion	59
Chapter 4: Clause Learning as a Proof System		60
4.1	Natural Proper Refinements of a Proof System	62
4.2	A Formal Framework for Studying Clause Learning	63
4.2.1	Decision Levels and Implications	63
4.2.2	Branching Sequence	64
4.2.3	Implication Graph and Conflicts	65
4.2.4	Trivial Resolution and Learned Clauses	67
4.2.5	Learning Schemes	69
4.2.6	Clause Learning Proofs	71
4.2.7	Fast Backtracking and Restarts	71
4.3	Clause Learning and Proper Natural Refinements of RES	72
4.3.1	The Proof Trace Extension	72
4.4	Clause Learning and General Resolution	74

4.5 Discussion	76
Chapter 5: Using Problem Structure for Efficient Clause Learning	78
5.1 Two Interesting Families of Formulas	80
5.1.1 Pebbling Formulas	80
5.1.2 The GT_n Formulas	81
5.2 From Analysis to Practice	82
5.2.1 Solving Pebbling Formulas	82
5.2.2 Solving GT_n Formulas	88
5.2.3 Experimental Results	90
5.3 Discussion	92
Chapter 6: Symmetry in Satisfiability Solvers	94
6.1 Preliminaries	97
6.1.1 Constraint Satisfaction Problems and Symmetry	98
6.1.2 Many-Sorted First Order Logic	98
6.2 Symmetry Framework and SymChaff	99
6.2.1 k -complete m -class Symmetries	100
6.2.2 Symmetry Representation	101
6.2.3 Multiway Index-based Branching	103
6.2.4 Symmetric Learning	104
6.2.5 Static Ordering of Symmetry Classes and Indices	105
6.2.6 Integration of Standard Features	105
6.3 Benchmark Problems and Experimental Results	106
6.3.1 Problems from Proof Complexity	107
6.3.2 Problems from Applications	108
6.4 Discussion	111
Chapter 7: Conclusion	114
Bibliography	116

LIST OF ALGORITHMS

Algorithm Number	Page
2.1 <code>DPLL-recursive</code> (F, ρ)	14
2.2 <code>DPLL-ClauseLearning</code>	17
3.1 <code>VC-greedy</code>	55
4.1 <code>DPLL-ClauseLearning</code>	63
5.1 <code>PebSeq1UIP</code>	84
5.2 <code>GridPebSeq1UIP</code>	86
5.3 <code>GTnSeq1UIP</code>	90

LIST OF FIGURES

Figure Number	Page
1.1 The three applications of proof complexity explored in this work	3
3.1 Viewing independent sets as a mapping	27
3.2 Toggling property of block-respecting independent sets	45
4.1 A conflict graph	66
4.2 Deriving a conflict clause using trivial resolution	68
4.3 Various learning schemes	69
4.4 Results: Clause learning in relation to resolution	77
5.1 A general pebbling graph	81
5.2 A simple pebbling graph	87
5.3 Approximate branching sequence for GT_n formulas	90
6.1 The setup for logistic planning examples	100
6.2 A sample symmetry file, <code>php-004-003.sym</code>	103
6.3 A sample PDDL file for <code>PlanningA</code> with $n = 3$	109

LIST OF TABLES

Table Number	Page
5.1 zChaff on <i>grid pebbling</i> formulas	91
5.2 zChaff on <i>randomized pebbling</i> formulas	91
5.3 zChaff on GT_n formulas	92
6.1 Experimental results on UNSAT formulas	111
6.2 Experimental results on SAT formulas	112